The Effect of Market Power on the Risk-free Rate*

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Abstract

We study how market power affects the risk-free rate when monopoly profits are capitalized into risky equity. Our key contribution is to show that the risk of capitalized profits is crucial for the effect of market power on interest rates. We develop a continuous-time model in which heterogeneous households face labor-income and equity-return risk and allocate wealth between safe debt and risky equity. Higher markups reduce exposure to labor-income risk but increase exposure to equity risk, generating opposing effects on precautionary demand for safe assets. We quantify the model to match key moments of the U.S. economy between 1984 and 2020: First, the model reproduces much of the decline in the risk-free rate, where the increase in exposure to risky equity plays a key role; second, rising market power contributes to 2.91 of the 5.43 percentage-points decline in the risk-free rate; third, household risk and heterogeneity amplify the welfare cost of rising markups: the consumption-equivalent loss is 14.51% in the full model, compared with 5.63% in a representative-household benchmark without risk. Our results show that the riskiness of capitalized profits is central to whether market power raises or lowers the risk-free rate and how large the welfare costs of markups are.

Keywords. Market Power. Financial Frictions. Risk-free rate. Welfare.

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1 Introduction

Since 1984, the U.S. economy has experienced a marked decline in the risk-free rate from 6.27 percent in 1984 to 0.03 percent in 2020. Understanding the mechanism behind this decline is of primordial importance for macroeconomic policy and welfare because the risk-free rate serves as the fundamental price for saving, investment and portfolio choice, as well as asset valuation and monetary policy. Alongside the decline in the risk-free rate, there is an upward trend in market power (Figure 1). Whether these two trends in markups and the risk-free rate are connected is an open question. In fact, existing theories of market power deliver opposite predictions; some theories imply that market power lowers the risk-free rate, while other theories imply that rising market power increases the risk-free rate. This difference in theoretical predictions is the starting point of our paper.

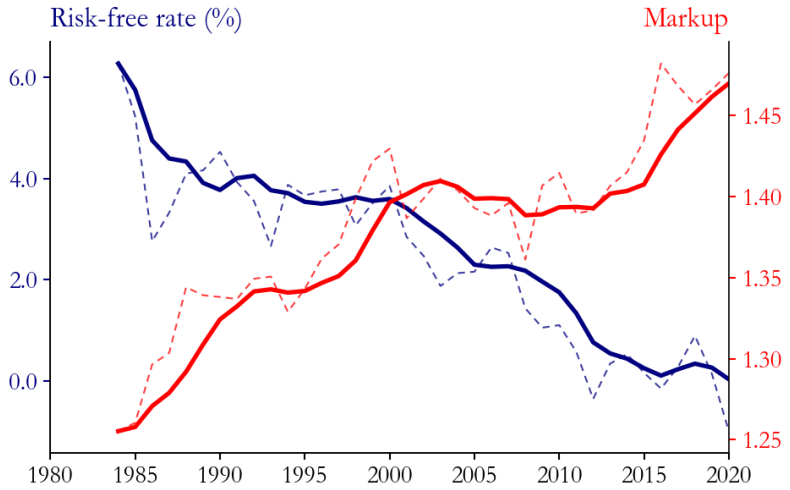


Figure 1: Markups and the real risk-free rate: 1984–2020.

Note: Cost-weighted markups computed following De Loecker et al. (2020), and the real risk-free rate is the 10-year US Treasury yield deflated by the Perceived Target Rate (PTR), an estimate of the Federal Reserve's perceived long-run inflation target. Thicker lines are the 5 year moving averages.

A priori, the effect of market power on the risk-free rate is ambiguous. In Eggertsson, Mehrotra, and Robbins (2019), Ball and Mankiw (2023) and Peruffo and Platzer (2024), market power lowers the risk-free rate by reducing investment (firms produce less when raising prices) and by reducing the supply of assets available to the households. On the other hand, Auclert and Rognlie (2018) and Auclert, Malmberg, Rognlie, and Straub (2025) show that this prediction reverses once future monopoly profits are capitalized into tradable equity and included in aggregate wealth. In this case, if labor income is risky and equity is riskless and therefore a *perfect substitute* for safe debt, market power increases the component of the household income that is safe. As a result, market power increases the share of safe income and reduces the share of risky labor income. Higher market power therefore lowers the risk households face, which lowers the demand for safe assets and pushes the risk-free rate upward.

We contribute to this debate by highlighting an important aspect of wealth. What matters is not only how much wealth higher market power creates for households, but also how risky those wealth

holdings are. We show that the riskiness of equity is crucial in determining whether the rise in market power increases or decreases the risk-free rate. If equity is subject to sufficiently high return risk, then capitalized profits no longer behave like safe wealth. A household with a large stock of equity sees its equity return risk exposure increased and generates a *complementary* demand for safe assets. This implies that in response to higher markups, households increase their demand for safe assets to hedge this additional risk, thereby putting downward pressure on the risk-free rate. The sign of the effect depends on how strongly higher markups increase households' exposure to risky equity relative to the reduction in their exposure to labor-income risk.

To formalize the key role played by risk in equity, we develop a continuous-time model of financial frictions with two types of assets, safe corporate debt and equity claims on (future) monopoly profits, building on Achdou, Han, Lasry, Lions, and Moll (2021) and Angeletos (2007). The heterogeneous households are subject to uninsurable labor income risk and equity-return risk and choose how to allocate their wealth between safe and risky assets. Borrowing constraints and costs of holding equity limit their ability to insure against these risks. As a result, the equilibrium risk-free rate is determined by the household Euler equations that incorporate precautionary motives and other wedges due to financial frictions, rather than by time preference alone. On the production side, firms have market power and generate monopoly profits, which are capitalized into equity. The firms can also issue safe corporate debt which is subject to a collateral constraint tied to firms' capital stock. Together, the debt and equity are the two sources through which a firm can finance its capital. The equilibrium risk-free rate is therefore jointly determined by the supply of safe assets and households' portfolio demand in response to labor and equity risk.

The central theoretical result of this paper is an equilibrium characterization of the risk-free rate that separates different forces: the standard intertemporal motive, precautionary savings due to labor and equity-return risk, consumption behavior of the hand-to-mouth households, and wedges due to equity holding costs and the leverage constraint. This characterization helps us to clearly isolate channels through which market power affects the risk-free rate and the key role that equity risk plays in determining the direction in which market power moves the risk-free rate.

Using the closed-form equilibrium equation for the risk-free rate, we illustrate this mechanism through two limiting cases. First, when households face no uninsurable labor and equity-return risk and no financial frictions, the returns on the safe debt and equity coincide and are determined by time preference and productivity growth. In this case, market power affects investment, output and welfare but not the risk-free rate. Second, consider the limiting case where labor income is risky but equity is riskless. In this case, safe debt and equity are perfectly substitutable stores of value. Higher market power now shifts income and wealth away from risky labor income toward safe capitalized profits. Households' exposure to risk falls, precautionary demand for safe assets declines, and therefore the risk-free rate increases. This case captures the mechanism emphasized by Auclert et al. (2025) and Auclert and Rognlie (2018). In the full model, equity itself can be risky along with risky labor income. Higher markups now generate two opposing forces. With higher average equity returns, the decline in households' exposure to labor-

income risk continues to put upward pressure on the risk-free rate. At the same time, the increase in risky equity wealth raises households' exposure to equity-return risk and therefore raises their demand for safe assets. When the latter force is sufficiently strong, the risk-free rate falls. The effect of market power therefore depends critically on the risk associated with the wealth created by higher markups. Therefore, *what matters is not only how much wealth higher markups create, but also how risky that wealth is.*

We quantify the model using U.S. data and assess the role of six structural forces in shaping the decline in the risk-free rate: i) markups, ii) fixed costs, iii) firms' collateral constraints on debt issuance, iv) equity-return risk, v) labor-income risk, and vi) productivity growth. The quantitative exercise allows us to measure not only how much each of these forces contributes to the decline in the risk-free rate, but also how changes in market power interact with household risk and portfolio choice.

The quantification yields three main findings. First, despite not targeting the risk-free rate, the model closely tracks its decline between 1984 and 2020. It reproduces both the relatively high risk-free rate of the 1980s and the subsequent fall to zero. This is an important validation of the mechanism. Second, our counterfactual exercises show that the increase in markups accounts for half of the decline in the risk-free rate. The mechanism is quantitatively driven by the change in household risk exposure highlighted by our theoretical analysis: higher markups, when the resulting profits are capitalized, reduce exposure to labor-income risk but increase exposure to risky equity, with the latter effect dominating through the precautionary motives. Therefore, the capitalization of monopoly profits into risky equity pushes the risk-free rate downwards once the risk associated with those claims is taken into account by the risk-averse households.

Third, market power has substantially larger welfare consequences in the presence of household heterogeneity and idiosyncratic risk. Our welfare counterfactual implies that the rise in markups between 1984 and 2020 changes welfare by -14.51%. When we instead consider the limiting case with a representative household and no risk, the welfare loss falls to -5.63%, close to the estimates in De Loecker et al. (2021) and Edmond et al. (2023). This comparison shows that the welfare cost of market power extends beyond the conventional losses from distorted production and lower investment associated with pure deadweight loss. By changing the composition and riskiness of household income and wealth, higher markups substantially amplify welfare losses. The novel insight here is that market power substantially increases the risk exposure of households.

Finally, we study the transitional dynamics after a permanent increase in markups. As flagged by Rachel (2025), accounting for transitions between steady states is important for understanding the evolution of asset returns. We consider both an unexpected permanent increase in markups and the announcement of the historical path of markups (1984-2020), solving for the perfect-foresight transitions. The adjustment is shaped by household heterogeneity along two dimensions. First, changes in wages, asset valuations, and returns affect aggregate consumption differently depending on households' marginal propensities to consume (Auclert, 2019). Second, the reallocation of risky equity across households depends on their marginal propensities to take risk (Kekre and Lenel, 2022). We find that the risk-free rate falls sharply upon announcement, as forward-looking equity valuations adjust immediately. The sub-

sequent adjustment is highly persistent, consistent with the slow convergence of wealth distributions under random growth highlighted by Gabaix et al. (2016). Nevertheless, despite a temporary increase in aggregate consumption, welfare falls immediately upon announcement and converges to its long-run level considerably faster than capital and asset holdings.

Related Literature. Understanding the mechanisms behind the sharp decline in the risk-free rate from the 1980s onward has captured the attention of academics and policymakers. There are many competing candidate explanations for this phenomenon, ranging from demographics (aging population, changing fertility) to technological change (productivity growth, changing capital intensity), government policy (debt, taxes and spending), the balance of payments, and market power... Several papers aim to account for the contributions of all these factors: Eggertsson, Mehrotra, and Robbins (2019); Peruffo and Platzer (2024); Rachel (2025); Auclert, Malmberg, Rognlie, and Straub (2025), among others. Here we focus exclusively on the role of market power in an attempt to isolate a transparent mechanism. That does not mean we believe that factors other than market power are unimportant conceptually and quantitatively.

Recent empirical work by Del Negro, Giannone, Giannoni, and Tambalotti (2017) directly measures the determinants of the decline in the risk-free rate. They highlight the role of the convenience yield, the compensation that households demand for holding more risky assets. They show that the convenience yield is on the rise, which results in a fall in the risk-free rate. This is consistent with our findings, where we analyze a mechanism that generates these facts, a lower risk-free rate and a higher premium on risky equity (higher convenience yield). Furthermore, Del Negro, Giannone, Giannoni, and Tambalotti (2019) show that the increase in the convenience yield is in large part a global phenomenon. While the focus of our analysis is the US economy, markups have risen worldwide (see De Loecker and Eeckhout (2018)), which lends credence to the fact that the mechanism we propose, driven by rising market power, can rationalize the fall in the risk-free rate and the rise in the convenience yield.[1]

Ball and Mankiw (2023) show that in the standard neoclassical model, market power drives a wedge between the marginal product of capital and the real rate, depressing it. Eggertsson, Mehrotra, and Robbins (2019) extend this insight into a quantitative overlapping generations model in which higher markups decrease capital demand, and find that this increase in markups has led to a decrease in the risk-free rate of 52 basis points between 1970 and 2015. Likewise, Peruffo and Platzer (2024) find that the rise in market power contributed to a 60 basis-point decline in the risk-free rate. A common feature of these three papers is that rents generated by market power accrue to households as current income or returns, rather than being capitalized into a financial asset.

In contrast, Auclert, Malmberg, Rognlie, and Straub (2025) and Auclert and Rognlie (2018) argue that a rise of market power does not lower the safe rate. Instead, an increase in markups raises the equilibrium return because it (i) boosts private asset *supply* through a higher capitalized value of profits and rents, and (ii) lowers the labor share – which is the only source of risk in their setting – scaling down

[1]The rise in the risk-free rate after the COVID pandemic does not necessarily fit the mirror image of the same mechanism. Del Negro, Elena, and Pham (2026) show that during this period there is no overwhelming evidence that the convenience yield has declined, nor is there evidence that market power has gone down. This suggests that the post-COVID rise in the risk-free rate is due to a mechanism that is distinct from the one we analyze here.

household *demand* for insurance. The key difference is that once profits are capitalized, households must be compensated for holding additional wealth.[2] Our model nests this comparative static once we shut down uncertainty in equity returns. However, we show that when profits are capitalized into risky assets, market power reduces the risk-free rate through a portfolio-rebalancing channel: households increase their demand for safe assets to hedge their portfolios against equity risk provided the risk is large enough.

Close to our framework, Farhi and Gourio (2018) propose a model that can explain trends in macro-finance such as the wedge between the marginal product of capital and the safe rate, the increase in the equity risk premium, and the rise in market power, among others. Their model incorporates aggregate disaster risk to generate changes in the equity premium. They find that the main drivers of the wedge between the marginal product of capital and the risk-free rate are increased markups and risk premia. Relative to their findings, our paper also incorporates an endogenous premium that is key for the determination of the safe rate. However, we model it as idiosyncratic return risk instead of aggregate shocks.[3]

Our model of idiosyncratic return risk builds on Angeletos (2007), who shows that uninsured investment risk generates a premium on private equity, even in the absence of aggregate uncertainty: higher uncertainty lowers the risk-free rate. We study how market power affects the capitalization of monopoly profits which affects the quantity of risky equity in the household budget, even if the underlying risk of equity returns does not change. Empirically, Fagereng et al. (2016) document substantial heterogeneity in household returns in Norway, including within narrow asset classes, and distinguish persistent return differences from time-varying idiosyncratic variation. Xavier (2021) provides complementary evidence for the United States, documenting return differences associated with both portfolio composition and heterogeneity within asset classes. These findings motivate allowing for household-level return heterogeneity, which our model captures in part through idiosyncratic equity-return risk.

Our model builds on work in heterogeneous-agent macroeconomics that studies how household saving and portfolio choice affect aggregate outcomes. Kaplan, Moll, and Violante (2018) emphasize differences in liquidity across assets, while Guvenen (2009) studies how limited stock-market participation and preference heterogeneity affect asset prices. Mian, Straub, and Sufi (2020) document the saving glut of the rich and its importance for safe asset demand, while Mian, Straub, and Sufi (2021) provide evidence that rising income inequality contributes to the decline in the natural rate of interest through differences in household saving rates. Mijakovic (2026) shows that rising income inequality also changes aggregate portfolio composition and equilibrium returns. In recent work, Cai and Zhang (2026) study how higher markups affect interest rates through changes in productive capital, redistribution toward business owners, and the capitalization of monopoly profits. In their one-asset benchmark, capitalized profits provide a safe store of value, so asset creation counteracts the downward pressure on interest

[2] One of the first papers to point out how profits capitalization and distribution of wealth is key in the heterogeneous agents class of models is Kaplan et al. (2018), although they focus on monetary policy effects.

[3] Rachel (2025) studies the risk-free rate without portfolio choice and observes that the risk premium spread has a profound impact on the risk-free rate and hence on policy.

rates from redistribution and lower productive capital. We show that when capitalized profits expose households to uninsurable equity-return risk, the additional wealth can itself increase precautionary demand for safe assets. Whether higher markups raise or lower the risk-free rate then depends on how strongly equity-risk exposure increases relative to the reduction in labor-income risk.

Recent work links wealth inequality to portfolio choice and declining interest rates. Cioffi (2021) develops a partial-equilibrium model in which non-homothetic preferences generate equity shares that increase with wealth, allowing aggregate return shocks and portfolio adjustments to explain substantial movements in U.S. wealth concentration. Greenwald et al. (2026) show that falling real rates raise wealth inequality because wealthy households hold more long-duration assets, with saving responses leaving these revaluation effects largely intact. Our paper shares their emphasis on heterogeneous portfolios but studies how market power changes household risk exposure and determines the equilibrium risk-free rate. The contributions are complementary: they explain inequality through returns and valuations, while we identify how rising market power can lower the safe rate through increased exposure to risky capitalized profits.

Our mechanism is one where the decline in the risk-free rate is driven by the rise in market power. Liu et al. (2022) instead argue that there is a reverse causal link from the risk-free rate to market power. Firms that have access to cheaper capital due to a lower risk-free rate invest more, which in turn affects the market power. The mechanism in their paper has leading firms investing more than following firms, and it is that heterogeneous access to investment that drives the effect on markups. This is a complementary mechanism that is absent in our paper. Instead, our households rebalance their portfolio in the light of the exposure to more equity in the household's budget. Despite the different mechanism, we find that market power and the risk-free rate move in opposite directions, consistent with Liu et al. (2022).

Our paper is also related to the literature on quantifying the welfare costs of rising market power. Our quantitative exercise is closest to those conducted in Edmond et al. (2023) and De Loecker et al. (2021). In limiting cases of our model without risk and precautionary savings, the welfare losses from higher markups are close to estimates in their models. Relative to these papers, we add household risk and risky equity, which considerably amplify the welfare losses as monopoly profits are capitalized into equity and households become more exposed to risk.

2 Model

Environment. Time is continuous and measured in years. The economy consists of a unit measure of households and a unit measure of intermediate goods producers. In each market j, a finite number N of firms compete strategically.[4]

Households are heterogeneous in financial wealth and idiosyncratic labor efficiency. Labor is supplied inelastically, and income depends on the wage per efficiency unit and individual productivity. Households choose consumption and allocate their financial wealth between safe and risky assets, mak-

[4]The measure of markets is $1/N$ such that each of the unit measure of intermediate goods producers are randomly assigned to a market with a total of N finite competitors.

ing portfolio choices continuously. Uncertainty is entirely idiosyncratic. Households face stochastic labor income risk and stochastic returns on equity. There is no aggregate uncertainty, so the economy admits a stationary distribution of households over wealth and productivity.

On the production side, a competitive firm aggregates inputs from intermediate-goods producers who operate under oligopolistic competition and issue equity and debt claims held by households. Prices in the economy are the wage, the risk-free return, and the expected return on equity.

Preferences. Preferences are represented by instantaneous CRRA utility

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \qquad \gamma > 0,\ \gamma \neq 1, \tag{1}$$

where γ denotes the coefficient of relative risk aversion. Future utility is discounted at the constant rate $\rho > 0$.

Labor Income Process. At each instant t, a household earns labor income equal to $w_t z_t$, where w_t denotes the wage per efficiency unit and z_t represents idiosyncratic labor productivity. Labor is supplied inelastically, so income risk arises solely from fluctuations in z_t, which follows a log Ornstein–Uhlenbeck process,[5]

$$d\tilde{z}_t = \mu_z(z_t)\, dt + \sigma_z(z_t)\, dW^z, \tag{2}$$

where

$$\mu_z(z_t) = \left[-\theta_z\big(\log z_t - \log \bar{z}\big) + \frac{1}{2}\sigma_z^2\right] z, \qquad \sigma_z^2(z_t) = (\sigma_z z_t)^2,$$

and where $\theta_z > 0$ governs the speed of mean reversion and σ_z the volatility of income shocks. Importantly, for our estimates we use the standard deviation of the stationary log income process $\nu = \sigma_z / \sqrt{2\theta_z}$.

Portfolio composition and asset returns. Households hold total financial wealth a_t. This wealth can be allocated between a safe asset $a_{S,t}$ and a risky equity asset $a_{R,t}$, so that total assets at time t are:

$$a_t = a_{S,t} + a_{R,t} \tag{3}$$

The safe asset yields the risk-free rate of return r_S.

We model the risky asset in the same reduced-form manner in which we model labor income and as is common in the literature on wealth dynamics (see for example Aiyagari (1994)). Production and profits at the firm level are deterministic (as is labor productivity), yet we assume a stochastic process for equity income at the level of the household (as we do with household-level labor income).[6] The risky

[5] In logs, the process is: $d\log \tilde{z}_t = -\theta_z\big(\log z_t - \log \bar{z}\big)\, dt + \sigma_z\, dW^z$.

[6] Modeling the stochastic process directly linked to the production process is challenging in the context of labor income. It is even more challenging for equity income because the source of equity risk is in part due to aggregate risk, in addition to idiosyncratic risk, some of which can be diversified through portfolio choice. However, there is ample evidence that households do face equity risk (see for example Xavier (2021)) and in our quantitative exercise we match the equity risk to what we observe in the data at the household level.

asset yields an idiosyncratic instantaneous realization that follows a Brownian motion with drift r_R and volatility σ_R:

$$d\tilde{r}_{R,t} = \mu_R(r_R)dt + \sigma_R dW_t^R \tag{4}$$

where $\mu_R(r_R) = r_R$ is the expected return on equity and is determined endogenously. Households choose the portfolio composition (a_{St}, a_{Rt}) taking prices (w_t, r_S, r_R) as given. The processes W_{Rt} and W_{zt} are independent, so equity risk and labor income risk are uncorrelated.[7,8] Angeletos (2007) introduces idiosyncratic return risk in a way similar to ours, with a $\sigma_R = 0.2$ and a robustness check of $\sigma_R = 0.4$, which is close to our estimation in Section 5.

Growth. Let X_t denote the labor-augmenting productivity. The economy experiences labor-augmenting technological growth at a constant rate $\frac{\dot{X}_t}{X_t} = g$. We express all quantities in efficiency units, letting output in efficiency units be $Y_t = \frac{\hat{Y}_t}{X_t}$, where $\hat{Y}_t$ is non-stationary output and Y_t is on the balanced growth path. So for example, aggregate consumption $\hat{C}_t$ grows at rate g, while aggregate consumption in efficiency units C_t is constant in our stationary economy.

Wealth Dynamics. Financial wealth evolves according to the household budget constraint:

$$\begin{aligned} da_t = da_{S,t} + da_{R,t} &= \left(w_t z_t + (r_S - g)a_{S,t} - \Psi(a_t, a_{R,t}) - c_t\right)dt + (d\tilde{r}_{R,t} - gdt)a_{R,t} \\ &= \underbrace{\left(w_t z_t + (r_S - g)a_t + (r_R - r_S)a_{R,t} - \Psi(a_t, a_{R,t}) - c_t\right)}_{\mu_a} dt + \underbrace{\sigma_R a_{R,t}}_{\sigma_a} dW_t^R \end{aligned} \tag{5}$$

The drift term μ_a reflects income — labor earnings conditional on current state z_t, the return on total wealth in the safe asset, and the expected return earned on risky holdings — net of consumption and financial services costs. The diffusion term captures equity return risk, which scales in proportion to risky asset holdings a_{Rt}.

The term $\Psi(a_t, a_{R,t})$ is the cost of holding the risky asset, which we model as follows:

$$\Psi(a_t, a_{R,t}) = \frac{\psi_0}{2}\frac{a_{R,t}^2}{a_t^{2+\psi_1}}, \qquad \psi_0 \geq 0, \quad \psi_1 \geq 0. \tag{6}$$

This cost, which we borrow from Gopalakrishna et al. (2026), is convex in risky asset holdings and decreasing in total wealth. The convexity captures the idea that larger equity positions are increasingly costly to maintain, while the dependence on total wealth makes equity participation relatively more expensive for low-wealth households. Economically, this reduced-form cost can be interpreted as capturing informational costs, transaction costs, participation frictions, or limited financial sophistication that make it harder for poorer households to access and manage risky portfolios. What these costs buy us in our model is a return on equity, net of costs, that increases with wealth, and a larger portfolio share

[7] This is approximately borne out in the US data (PSID) where the correlation coefficient is 0.06.

[8] The shocks dW_t^R are independent across households and cannot be insured, since no claim contingent on them is traded. These shocks cancel in the aggregate (see the asset-market clearing condition in Section 3.3).

of risky assets for the wealthy. The parameter ψ_0 governs the overall level of the cost, while ψ_1 controls how strongly the cost declines with wealth.[9]

Financial Constraints. Households face a borrowing constraint on the safe asset, together with a no-shorting constraint on the risky asset

$$a_{S,t} \geq \underline{a}, \qquad a_{R,t} \geq 0 \tag{7}$$

These constraints imply a constraint on households' leverage

$$0 \leq a_{R,t} \leq a_t - \underline{a} \tag{8}$$

When $\underline{a} = 0$, households cannot borrow and therefore cannot take leveraged positions in the risky asset. For the numerical solution, we will work with the equivalent state constraint $a_t \geq \underline{a}$ together with the leverage constraints.

Hamilton–Jacobi–Bellman Equation. Let $V(a, z)$ denote the value function of a household with financial wealth a and idiosyncratic labor productivity z. We are going to focus on the stationary economy, so we drop the subscript t. Taking prices (w, r_S, r_R) as given, the household chooses consumption c and risky asset holdings a_R to maximize lifetime utility. The stationary Hamilton–Jacobi–Bellman equation is:

$$[\rho + (\gamma - 1)g]\, V(a,z) = \max_{c \geq 0,\ a_R \in [0, a - \underline{a}]} \left\{ u(c) + V_a \mu_a(a,z;c,a_R) + \frac{1}{2} V_{aa} \sigma_R^2 a_R^2 + V_z \mu_z(z) + \frac{1}{2} V_{zz} \sigma_z^2 z^2 \right\} \tag{9}$$

The term $u(c)$ represents instantaneous utility from consumption. The term $V_a \mu_a$ captures the effect of expected changes in financial wealth on the continuation value. The term $\frac{1}{2} V_{aa} \sigma_R^2 a_R^2$ captures the curvature effect of equity return risk. Because equity returns are stochastic with volatility σ_R, fluctuations in wealth scale proportionally with risky holdings a_R, and the second derivative V_{aa} governs the precautionary response to this risk. The remaining terms, $V_z \mu_z(z)$ and $\frac{1}{2} V_{zz} \sigma_z^2 z^2$, arise from the stochastic evolution of labor productivity. These terms reflect the impact of expected changes and volatility in idiosyncratic labor income on the continuation value of the household.

The household chooses consumption c and the asset composition $\{a_S, a_R\}$ to maximize the discounted value $V(a, z)$ given current assets a and income shock z, subject to the leverage constraint (8) and with the state boundary enforcing $a_t \geq \underline{a}$. Since safe assets are just total assets minus risky assets: $a_S = a - a_R$, they are determined by the solution for a_R given states (a, z).

Technology. We model the production technology following the setup by Jaimovich and Floetotto (2008), where a representative firm produces a homogeneous final consumption good that uses as inputs all the goods produced in the continuum of input markets. The final goods market is competitive and frictionless. There is a continuum of input markets indexed by j and with measure $1/N$. In each

[9] In the computational implementation, we impose a floor on a_t in the denominator of equation (6).

market, there is a finite number of N firms, each indexed by n. The final good aggregates inputs with a double-nested Constant Elasticity of Substitution (CES) technology:[10]

$$Y_t = \left[\int_0^{1/N} Y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \quad Y_{jt} = \left[\sum_{i=1}^{N} N^{-\frac{1}{\eta}} Y_{njt}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{10}$$

The elasticities of substitution θ and η indicate the substitutability of the inputs in production of the final good, with $\theta < \eta$; goods within a market are closer substitutes than goods in different markets.[11] The final goods firms choose Y_{nj} to maximize profits subject to the budget constraint.

The representative firm in the input market uses a constant-returns-to-scale Cobb-Douglas technology with labor-augmenting Total Factor Productivity (TFP) $X_{njt} = X_t$ and with labor and capital as inputs of production:

$$\hat{Y}_{njt} = \hat{K}_{njt}^{\alpha}(X_t L_{njt}^P)^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{11}$$

or in efficiency units $Y_{njt} = K_{njt}^{\alpha}\left(L_{njt}^P\right)^{1-\alpha}$. Each period, firms pay a flow fixed (overhead) cost $w_t\phi_t$, expressed in overhead labor units ϕ_t/N. Because firms are identical in productivity and operate in identical markets, we drop the nj subscript throughout the paper when there is no ambiguity.

There is a spot market for labor L_t with price w_t in efficiency units. Firms need to invest to accumulate capital K_t to produce. Capital increases with investment I_t and depreciates at rate δ. We assume firms cannot divest. The law of motion of capital accumulation is:

$$dK_t = [I_t - (\delta + g)K_t]\, dt. \tag{12}$$

Market Structure and Conduct. The number of firms within an input market j is finite, so there is strategic interaction within the market.[12] There is Cournot competition, and we solve for the static Nash equilibrium.[13] To model variation in the extent of competition, we introduce common ownership to specify conduct. With common ownership, competing firms take into account the profits of the competitors in which they have an ownership stake when making production decisions (Rotemberg (1984); Azar (2012); Appel, Gormley, and Keim (2016); Ederer and Pellegrino (2025); Azar and Vives (2021)). In our Jaimovich and Floetotto (2008) setting, a firm n in a market j owns an equal fraction $\Omega \in [0,1]$ of the profits of the other firms $m \neq n$ in the same market. The common ownership parameter is identical across all firms and markets, and is time-varying: Ω_t.[14]

The objective of the firm is to maximize $\Pi_{njt} + \Omega_t \sum_{m\neq n} \Pi_{mjt}$, where $\Omega = 0$ is the Cournot solution

[10] This setup is closely related to that in Atkeson and Burstein (2008) where households aggregate differentiated goods produced in oligopolistic markets, that is, in Atkeson and Burstein (2008) the aggregation happens by household preferences instead of by the final goods producer here and in Jaimovich and Floetotto (2008).

[11] We introduce $N_j^{-\frac{1}{\eta}}$ to neutralize the love-of-variety effect.

[12] With a continuum of markets, there is no strategic interaction between markets.

[13] We assume that input firms match with new competitors continuously to avoid dynamic strategies of collusion.

[14] With identical firms, we could represent the economy with a monopolistically competitive production economy, where the markup would vary with the demand elasticity. However, the welfare implications of markups varying through preferences are fundamentally different from those through market structure or conduct as in our setup.

with N firms and $\Omega = 1$ corresponds to perfect collusion, or equivalently, monopoly pricing. We solve for the allocation in which each firm n, j chooses y_{njt} while taking as given the optimizing behavior of its competitors, the inverse demand p_{njt}, and the aggregate prices w_t, $r_{S,t}$, $r_{R,t}$, p_t.

Debt, Equity and Collateral. Firms have a choice to finance capital K_t with debt or with equity. Denote by D_t the stock of outstanding debt. The debt (or bond) is safe, and interest is serviced continuously at the risk-free safe rate r_S, so the flow cost is $r_S D_t$. Denote new borrowing by B_t:

$$dD_t = [B_t - gD_t]\, dt. \tag{13}$$

Risky assets generate dividend flows. The flow-of-funds identity is:

$$DIV_t\, dt = \left(p_t Y_t - w_t L_t^P - w_t \phi - I_t - r_S D_t + B_t\right) dt \tag{14}$$

Dividends equal revenue minus labor costs, investment, and interest payments plus net new borrowing.

Firms face financial frictions. They can borrow up to a fraction $\lambda \in [0, 1]$ of their capital stock K_t. With continuously serviced debt, the natural collateral restriction is on the outstanding debt stock:

$$D_t \leq \lambda K_t, \qquad \lambda > 0. \tag{15}$$

This is the continuous-time counterpart of discrete-time constraints written in terms of principals.[15]

Profits. We define a firm's operating profits (accounting for the cost of labor but not the user cost of capital and overhead) as:

$$\Pi_t^o = p_t Y_t - w_t L_t^P \tag{16}$$

The firm's accounting profits also adjust for debt payments, cost of capital, and overhead

$$\Pi_t^a = p_t Y_t - w_t L_t^P - w_t \phi - r_S D_t - \delta K_t \tag{17}$$

Economic profits also take into account the opportunity cost of holding equity:

$$\Pi_t = p_t Y_t - w_t L_t^P - w_t \phi_t - [r_{S,t}\lambda + (1-\lambda) r_{R,t} + \delta]\, K_t. \tag{18}$$

Firm Hamiltonian. The firm chooses labor L_t, investment I_t, and net borrowing B_t to maximize the present value of dividend payments, taking prices (w_t, r_S, r_R) as given:[16]

$$\max_{\{L_t^P, I_t, B_t\}} \int_0^\infty e^{-(r_R - g)t}\, DIV_t\, dt. \tag{19}$$

[15] Discrete-time forms like $(1 + r_S)D_t \leq \lambda K_t$ correspond to one-period maturity with gross repayment; in continuous time, the analog would involve $e^{r_S \Delta}$ for maturity Δ.

[16] Debt does not mature and is rolled over continuously

The firm is subject to capital accumulation, debt accumulation, and the collateral constraint:

$$dK_t = [I_t - (\delta + g)K_t]\, dt, \tag{20}$$

$$dD_t = [B_t - gD_t]\, dt, \tag{21}$$

$$D_t \leq \lambda K_t. \tag{22}$$

To characterize the optimal policy, we write the firm's problem in current-value form. Let q_t denote the shadow value, or current-value costate variable, associated with installed capital K_t. Let m_t^D denote the shadow value, or current-value costate variable, associated with outstanding debt D_t. Finally, let $\xi_t \geq 0$ denote the multiplier on the collateral constraint. Thus, q_t and m_t^D measure the marginal value of relaxing the capital and debt accumulation equations, while ξ_t measures the marginal value of relaxing the collateral constraint.

The current-value Hamiltonian is

$$\mathcal{H}_t = \Pi_t^o - w_t\phi - I_t - r_S D_t + B_t + q_t\left[I_t - (\delta + g)K_t\right] + m_t^D(B_t - gD_t) + \xi_t\left(\lambda K_t - D_t\right). \tag{23}$$

where we plugged in the definition of dividends from the flow-of-funds constraint. This Hamiltonian summarizes the firm's intertemporal trade-offs between investment, borrowing, and dividend payments.

Discussion of Assumptions

Ours is a framework to study household consumption and savings in the presence of portfolio choice and market power and general equilibrium price determination (interest rates, wages and goods prices). We have chosen to embed these choices in the context of a continuous-time savings, consumption and production decision, with the aim of obtaining full heterogeneity at the household level. While modeling this heterogeneous agent, continuous-time setting is challenging in its own right and we have made remarkable progress in that direction, we also face a tradeoff in modeling choices to keep this model tractable and obtain analytical and computational solutions. Next, we discuss the different assumptions and modeling choices in the light of this tradeoff.

Idiosyncratic vs Aggregate Risk. In our setup, we only have idiosyncratic risk. Labor income and especially equity income are also subject to aggregate fluctuations. However, introducing aggregate fluctuations that directly link to equity returns in the presence of full heterogeneity is computationally more challenging, as the infinite-dimensional distribution becomes part of the economy's state. Moreover, because this paper focuses on trends in the US economy, adding business cycle shocks is not central. Instead, for tractability, we abstract from aggregate fluctuations completely and model the equity return process as a reduced-form, idiosyncratic risk process at the level of the household. Note that we maintain the same assumption for labor income risk, which is common in the literature: firms generate deterministic output, yet workers receive stochastic labor income. It is worth noting that even for equity income,

there is ample evidence of idiosyncratic returns. Fagereng et al. (2016) and Xavier (2021) show that equity returns (especially private equity) are heterogeneous, despite the fact that households can diversify portfolios. Farhi and Gourio (2018) consider aggregate shocks, but do not include heterogeneity. In our setting, heterogeneity allows us to have an endogenous premium with standard preferences. In our baseline model we quantify the steady state in each year. In section 5.6 we also analyze the transitional dynamics.

The Cost of Holding Equity and Homothetic Preferences. One well-known (and counterfactual) feature of portfolio choice models is that they tend to generate too much equity exposure at the bottom of the wealth distribution. Angeletos (2007) provides a useful benchmark: homothetic preferences and linear returns imply a constant portfolio share of risky assets relative to effective wealth. This effective wealth includes relatively safer human wealth, which implies very high risky shares in financial wealth at the bottom of the distribution. The cost Ψ in equation (6), which we borrow from Gopalakrishna et al. (2026), makes it proportionately more expensive for poor households to hold equity and mechanically yields an equity share that matches the data. Guvenen (2009) has a restriction on stock market participation which is similar in spirit to our cost of holding equity. Alternative approaches introduce non-homothetic preferences so savings and risk-taking behavior vary with wealth. For example, Straub (2019) shows that this non-homotheticity can explain high saving rates of the rich (see also Cioffi (2021), Allais et al. (2020), and Mijakovic (2026)). Such preferences can feature decreasing absolute risk aversion (DARA) and generate larger risky portfolios at the top of the wealth distribution, but we find that they do not fix the excessive risk-taking at the bottom.

Absence of life cycle. An alternative approach to modeling an elastic demand for assets and thus moving the safe rate is to include overlapping generations. Life cycle motives, such as demographics and bequests, play an important role in determining rates (Auclert et al., 2025), but they are not the focus of our investigation. Nevertheless, we expect our results on welfare and rates to survive in such a setup. Intuitively, the life cycle introduces another saving motive: retirement. Differences in age will generate differences in risky-asset portfolio shares, with households closer to retirement usually willing to bear less risk. Thus, a rise in market power will change the supply of risky assets and still require households to absorb it. This can generate the same portfolio-rebalancing channel that can increase demand for safe assets and push the risk-free rate down, while the increase in risky exposure is as likely to hurt welfare as in our setup.

Firm Collateral Constraint. Our firms are constrained by the amount of assets they can put up as collateral (similar to the collateral constraint that households face in Kiyotaki and Moore (1997)). Recent work by Li, Lian, Ma, and Martell (2025) and Kermani and Ma (2020) instead uses earnings-based constraints, consistent with the evidence that firms with higher profits borrow against their projected future earnings. In our quantification with homogeneous productivity across firms and frictions based

on collateral constraints, we find that changes in the constraint play a secondary role in determining the rate.

Identical Firms. The model incorporates a rich setup with oligopolistic competition (Cournot) that allows for variation in market structure (number of competitors) and conduct (common ownership). For tractability we assume firms are identical and have the same productivity. Firm heterogeneity plays an important role for understanding the distribution and evolution of markups (De Loecker et al. (2020), Edmond et al. (2023), De Loecker et al. (2021)) as well as the contribution of reallocation of sales shares towards high-markup firms. However, our focus here is on the household consumption-savings decision, and marrying full heterogeneity at both the household and the firm level is a challenge. We have therefore opted to allow for rich market structure and conduct, while abstracting from firm heterogeneity.

Cournot vs Bertrand Competition. The firms in our model compete à la Cournot in quantities in markets with differentiated goods. We can allow for Bertrand competition in differentiated goods instead, which yields a more competitive solution and a less dispersed distribution of markups. Since our firms are identical, the latter has no effect in the model. For that reason, the results under Bertrand are virtually identical to those under Cournot.

Other forces: Demographics, Government Debt, Social Security, Foreign Demand for US Assets. Our model abstracts from several other forces that have been shown to matter for the long-run risk-free rate. These include demographics, government debt, social security, and foreign demand for U.S. assets. These structural forces feature prominently in Eggertsson et al. (2019), Peruffo and Platzer (2024), and more recently in Auclert et al. (2025), who quantify their contributions to movements in equilibrium interest rates. Incorporating them in a quantitatively meaningful way would require adding a life-cycle component, as well as fiscal and monetary policy, and potentially an open-economy structure to our framework. We abstract from these forces in order to isolate the interaction between market power, portfolio choice, and household risk and in order to highlight the key mechanisms at work using a closed-form expression of the equilibrium risk-free rate.

The period 1984-2020. Our period of analysis for the quantitative exercise covers 1984 to 2020, which is also the period in which there is a decline in the risk-free rate. The years prior to 1984 were a period of high inflation with the transition towards inflation targeting with high federal funds rates under the mandate of Paul Volcker as the Federal Reserve Chair. In that period forces different from the ones we study here were likely at play. And post 2020, we had again high inflation due to the COVID recession.

3 Solution

3.1 Households' solution

Optimal Investment Policies. The household chooses consumption c, and safe and risky assets a_S, a_R. With $V_a > 0$ and $V_{aa} < 0$ (concavity in a), then consumption solves the FOC $u'(c) = V_a$ which gives

$$c^*(a,z) = V_a(a,z)^{-1/\gamma}. \tag{24}$$

The optimal asset position satisfies the FOC in a_R

$$(r_R - r_S)V_a - \psi_0 \frac{a_R}{a^{2+\psi_1}} V_a + \sigma_R^2 a_R V_{aa} - \zeta(a,z) + \iota(a,z) = 0. \tag{25}$$

where $\zeta(a,z)$ and $\iota(a,z)$ are the Lagrange multipliers associated with the leverage constraint (8), where the former relates to the upper bound $a_R \leq a_t - \underline{a}$ and the latter to the lower bound $a_R \geq 0$. It can be shown that, for a positive premium $r_R - r_S > 0$, $\iota(a,z) = 0$, that is, the no-shorting constraint will never bind. Solving for the interior risky investment condition, we get:

$$\sigma_R \, a_R^\star(a,z) = \underbrace{\frac{r_R - r_S}{\sigma_R}}_{\text{Sharpe Ratio}} \underbrace{\left(-\frac{V_{aa}(a,z)}{V_a(a,z)}\right)^{-1}}_{\text{Risk Tolerance}} \underbrace{\left[1 + \frac{\psi_0}{\sigma_R^2 \, a^{2+\psi_1}} \left(-\frac{V_a(a,z)}{V_{aa}(a,z)}\right)\right]^{-1}}_{\text{Equity Holding Cost}}. \tag{26}$$

This equation states that the *optimal unconstrained risky exposure* equals the Sharpe ratio times the inverse of the Absolute Risk Aversion coefficient, which can be interpreted as risk tolerance. The term in brackets accounts for the fact that investors also need to be compensated for the cost of holding equity, and it equals one if that cost is zero. With the leverage constraint, the optimal policy function can be written as:

$$a_R(a,z) = \min\Big\{ \max\{a_R^*(a,z), 0\}, \, a - \underline{a} \Big\}. \tag{27}$$

In the frictionless CRRA utility Merton (1969) problem without labor-income risk, the optimal risky *share* of wealth is constant:

$$\frac{a_R}{a} = \frac{r_R - r_S}{\gamma \sigma_R^2}. \tag{28}$$

In our setting, as $a \to \infty$, the policies converge toward this benchmark (income becomes negligible relative to wealth), which motivates the use of tail boundary conditions in numerical work.

Stationary Kolmogorov Forward Equation. Let $m(a,z)$ be the stationary joint density on the domain $\mathcal{A} = [\underline{a}, a_{\max}] \times [z_{\min}, z_{\max}]$ (or suitable truncation). The stationary KFE is the adjoint of the infinitesimal generator:

$$0 = -\partial_a\big(\mu_a(a,z)\, m(a,z)\big) + \tfrac{1}{2}\partial_{aa}\big(\sigma_a(a,z)^2\, m(a,z)\big) - \partial_z\big(\mu_z(z)\, m(a,z)\big) + \tfrac{1}{2}\partial_{zz}\big(\sigma_z(z)^2\, m(a,z)\big), \tag{29}$$

with normalization $\int_{\mathcal{A}} m(a,z)\,\mathrm{d}a\,\mathrm{d}z = 1$.

3.2 Firm Solution

Demand and Pricing. The intermediate good firm's static optimization solves the first-order condition with respect to Y_{nj} is (for the full derivation, see Appendix A):

$$\frac{\partial DIV_{njt}}{\partial Y_{njt}} + \Omega_t \sum_{m \neq n} \frac{\partial DIV_{mjt}}{\partial Y_{njt}} = 0 \tag{30}$$

Moreover

$$\frac{\partial p_{mj}}{\partial Y_{nj}} Y_{mj} = \left(\frac{1}{\eta} - \frac{1}{\theta}\right) s_{nj} \frac{s_{mj}}{s_{nj}} p_{nj} \tag{31}$$

and where $s_{nj} = \frac{1}{N}$ is firm n's sales share in market j. Dropping the nj subscripts, the FOC can be written as

$$p_t \left[1 - \frac{1}{\eta} + \left(\frac{1}{\eta} - \frac{1}{\theta}\right)\frac{1}{N}\Big(1 + \Omega_t(N-1)\Big)\right] = w_t \frac{\partial L_t}{\partial Y_t} + \left[\lambda r_{S,t} + (1-\lambda) r_{R,t} + \delta\right] \frac{\partial K_t}{\partial Y_t} \quad \Leftrightarrow \quad p_t \underbrace{\left(1 + \varepsilon_t^p\right)}_{\mu_t^{-1}} = MC_t \tag{32}$$

where MC_t is the marginal cost which we will derive in next section, ε_t^p is the residual demand elasticity, and the markup is given by:

$$\mu_t = \frac{p_t}{MC_t} = \left[1 - \frac{1}{\eta} + \left(\frac{1}{\eta} - \frac{1}{\theta}\right)\frac{1}{N}\Big(1 + \Omega_t(N-1)\Big)\right]^{-1} \tag{33}$$

Firm optimality conditions. In Appendix A, we derive the following expressions from the firm's current-value Hamiltonian. The firm's static labor demand equates the wage to the marginal revenue product of labor. Given our market structure, marginal revenue is equal to price divided by the gross markup μ, so

$$w_t = \frac{p_t}{\mu}(1-\alpha) K_t^{\alpha} \left(L_t^P\right)^{-\alpha}. \tag{34}$$

The capital Euler equation equates the full user cost of capital to the marginal benefit of capital. In steady state, with a positive premium ($r_R - r_S > 0$) this condition is

$$r_R + \delta = \frac{p_t}{\mu} \alpha K_t^{\alpha - 1} \left(L_t^P\right)^{1-\alpha} + \lambda(r_R - r_S). \tag{35}$$

The first term on the right-hand side is the marginal revenue product of capital, while the second term is the collateral value of capital.

Steady-state capital and balance sheet. Under the normalization $p = 1$ and fixed labor supply $L = 1$, the capital Euler equation implies

$$K = L^P \left[\frac{\frac{1}{\mu}\alpha}{\lambda r_S + (1-\lambda) r_R + \delta} \right]^{\frac{1}{1-\alpha}}. \tag{36}$$

Thus, market power lowers the productive return to capital through the term $1/\mu$, while collateralizable capital lowers the effective user cost when $r_R > r_S$. The corresponding wage is

$$w = \frac{1}{\mu}(1-\alpha) \left(\frac{K}{L^P} \right)^{\alpha}. \tag{37}$$

When the collateral constraint binds, the firm borrows up to the collateral limit,

$$D = \lambda K, \qquad E = K - D = (1-\lambda)K. \tag{38}$$

where E is book equity, the part of capital financed with internal funds (retained earnings).

3.3 Market Clearing

Households solve the portfolio choice problem given prices (r_S, r_R, w), generating policies $c(a,z)$, $a_R(a,z)$, and a stationary density $m(a,z)$.

Asset market clearing We interpret the unique risky asset held by households as the *market portfolio of equity*. Then aggregate household demand for risky dollars equals the market value of equity: [17]

$$\iint a_R(a,z)\, m(a,z)\, da\, dz \;=\; V = \frac{DIV}{r_R - g} = E + \frac{\Pi^e}{r_R - g}.$$

where E and Π^e are the book value of equity and economic profits aggregated across firms. Risk-free asset clearing equates aggregate household safe holdings to the supply of risk-free claims. If corporate debt is the only risk-free claim held by households, then

$$\iint \big(a - a_R(a,z)\big)\, m(a,z)\, da\, dz \;=\; \iint a_S(a,z)\, m(a,z)\, da\, dz \;=\; D.$$

Labor market clearing. Since labor is supplied inelastically, our aggregate labor supply is given by $L^S = \iint z\, m(a,z)\, da\, dz$. We normalize units so that $L^S = 1$, which pins down the scale of z (or equivalently the normalization of the stationary distribution).[18] On the demand side, each intermediate producer chooses labor L_{njt} given wages and its capital stock. Aggregate labor demand is the integral

[17] Clearing holds in market value, in efficiency units, and is consistent with deterministic firms. The shocks dW_t^R in (4) are independent across households and cancel in the aggregate, so equity holders as a group earn the firms' payout, $(r_{R,t} - g)V_t = \mathrm{DIV}_t$, where $r_R = \mu(r_R)$ in equation (4). For an individual household, $\sigma_R a_{R,t}\, dW_t^R$ is an uninsurable zero-sum transfer among equity holders: the return on the household's own portfolio, not on the aggregate market.

[18] With this normalization, the wage share coincides with the labor share.

over firms, which use labor to produce and pay overhead costs

$$L^D = \int_0^{1/N} \sum_{n=1}^{N} (L^P_{njt} + \phi)\, dj,$$

which equals L_t in a symmetric equilibrium, with homogeneous firms. Labor market clearing requires

$$L^D \;=\; L^S \;=\; 1.$$

Goods market clearing. The resource constraint is

$$C_t + I_t + \Psi_t = Y_t, \qquad dK_t = [I_t - (\delta + g)K_t]\, dt,$$

where $\Psi = \int\int \Psi(a,z) m(a,z)\, da\, dz$ is the aggregated cost of holding equity that households pay. In a steady state ($dK_t = 0$), goods market clearing becomes:

$$C + (\delta + g)K + \Psi = Y. \tag{39}$$

3.4 Market Power and the Risk-free Rate

Once we have solved the model for the policy functions on consumption $c(a,z)$ and equity $a_R(a,z)$ and obtained the stationary distribution $m(a,z)$ from the Kolmogorov Forward Equation, we can characterize r_S as the rate at which households can bring consumption into the future that satisfies stationarity of aggregate consumption in efficiency units $\frac{\mathrm{d}C}{\mathrm{d}t} = 0$. To do so, in Appendix A we derive the expected consumption drift for each individual household $\frac{E[\mathrm{d}c|a,z]}{\mathrm{d}t}$, and aggregate them to get the stationary aggregate consumption drift in efficiency units:

$$\begin{aligned} 0 = \frac{dC}{dt} &= \iint \frac{E[dc \mid a,z]}{dt}\, m(a,z)\, da\, dz \\ &= \iint_U \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz \;+\; \iint_L \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz \;+\; \iint_C \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz \end{aligned}$$

where in the last equality we divide households into constrained and unconstrained groups. Next, we plug in the derived expected consumption drifts and define $\widetilde{E}^S$ as the consumption-weighted average over group S. Constrained households' consumption depends exclusively on their efficiency units drift and real wages [19], while unconstrained households choose their optimal policies subject to labor and

[19] We use $\eta(z) \equiv \frac{\mu_z(z)}{z}$. If constrained households consume wage income $c = wz$ (hand-to-mouth), then $dc = w\, dz$ so

$$\frac{E[dc \mid a,z]}{dt} = w\mu_z(z) = (wz)\frac{\mu_z(z)}{z} = c(a,z)\,\eta(z) \qquad \text{for } (a,z) \in C.$$

equity risks. Together:

$$0 = \underbrace{\frac{1}{\gamma}\frac{C^U + C^L}{C}(r_S - \rho - \gamma g)}_{\text{Intertemporal substitution}} + \underbrace{\frac{C^U + C^L}{C}\frac{\gamma+1}{2}\widetilde{E}^{U\cup L}\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right]}_{\text{precautionary motive}} +$$
$$+ \underbrace{\frac{C^C}{C}\widetilde{E}^C[\eta(z)]}_{\text{mean drift of constrained}} - \underbrace{\frac{1}{\gamma}\frac{C^U + C^L}{C}\widetilde{E}^{U\cup L}[\Psi_a(a, a_R)]}_{\text{wealth-relaxation effect on equity costs}} + \underbrace{\frac{1}{\gamma}\frac{C^L}{C}\widetilde{E}^L\left[\frac{\zeta(a,z)}{V_a(a,z)}\right]}_{\text{upper-corner shadow value } (a_R \le a)} \tag{40}$$

Equation (40) decomposes the stationary aggregate consumption drift, which must be zero in steady state, into three terms that reflect the expected drift of each type of household's optimal consumption: (i) unconstrained households' intertemporal substitution weighted by their share in aggregate consumption; (ii) the precautionary savings motive generated by idiosyncratic risk arising from either labor efficiency units risk or equity returns risk, with the same weight; (iii) the expected drift in consumption of constrained households, weighted by their share of aggregate consumption; this drift is generally positive as constrained households are ''Hand-to-mouth" and their income is expected to revert to the mean. When constrained households (who are tied more closely to current income) tend to sit in states with higher expected mean reversion, their average drift exceeds that of the unconstrained, so the economy requires the unconstrained sector to absorb a larger predictable drift imbalance in order to keep aggregate consumption stationary. That is, in the stationary equilibrium, the risk-free rate r must be such that these three forces must offset each other. Solving for the risk-free rate we get the following equation:

$$r_S = \rho + \gamma g - \gamma \cdot \frac{\gamma+1}{2}\widetilde{E}^{U\cup L}\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right] - \gamma\frac{C^C}{C^U + C^L}\widetilde{E}^C[\eta(z)] - \chi^L - \chi^\Psi \tag{41}$$

where $\chi^L = \frac{C^L}{C^U + C^L}\left(\widetilde{E}^U\left[\sigma_R^2 a_R\,\gamma\left(\frac{c_a}{c}\right) + \psi_0\frac{a_R}{a^{2+\psi_1}}\right] - \widetilde{E}^L\left[\sigma_R^2 a_R\,\gamma\left(\frac{c_a}{c}\right) + \psi_0\frac{a_R}{a^{2+\psi_1}}\right]\right)$ is the consumption-weighted aggregate shadow value of the leverage constraint and $\chi^\Psi = \tilde{E}^{U\cup L}[-\Psi_a(a, a_R)]$ is the equity cost relaxation effect of an extra unit of wealth.

Equation (41) shows that departures of the safe rate from the time-preference benchmark arise through three distinct channels. First, household-level consumption risk generates a precautionary saving motive, captured by the variance of consumption growth among unconstrained and leverage-constrained households. Second, the presence of hand-to-mouth households, whose consumption is tied to current labor income, introduces a predictable drift component in aggregate consumption that must be offset in equilibrium. Third, leverage-constrained households distort the mapping between risk premia and aggregate consumption dynamics: because these agents are forced to hold corner portfolios, their marginal rate of substitution differs from that of interior traders, giving rise to a shadow-value wedge that reflects the tightness of the portfolio constraint. Fourth, both unconstrained and leverage-constrained households have to pay a relatively lower cost of holding equity as their wealth increases — they get a larger average net return. In the knife-edge case with no idiosyncratic risk, no holding costs, and no con-

strained households (so $\sigma_z = \sigma_R = \psi_0 = 0$ and $C^C = C^L = 0$), all wedges vanish and the stationary equilibrium reduces to $r_S = \rho + \gamma g$.

Risk and the precautionary wedge. The first wedge is a consumption-weighted average of the instantaneous variance of consumption growth faced by unconstrained households. It combines labor-income risk and equity-return risk,

$$\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2 \quad \text{and} \quad (\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2,$$

where $\frac{c_z}{c}$ and $\frac{c_a}{c}$ measure how strongly optimal consumption responds to shocks in productivity and wealth at a given state (a, z). The factor $(\gamma + 1)$ captures prudence: higher curvature raises the strength of precautionary saving for a given amount of diffusion risk. Since this wedge enters with a negative sign, greater exposure to either source of risk lowers the equilibrium risk-free rate required to keep aggregate consumption stationary.

Wealth-constrained agents and the drift wedge. The second wedge reflects predictable movements in constrained households' consumption induced by the drift in their labor income process. Under the hand-to-mouth boundary rule $c = wz$, constrained consumption inherits the conditional mean reversion of z through $\eta(z) = \mu_z(z)/z$. When constrained households are disproportionately located in states with positive expected income growth (e.g. low z states under mean reversion), the consumption-weighted average $\widetilde{E}^C[\eta(z)]$ is positive. The scaling factor $\frac{C^C}{C^U + C^L}$ then maps this predictable drift in the constrained block into the amount of offsetting adjustment required from the unconstrained and Leverage-constrained blocks; in equilibrium, this adjustment operates through a lower r_S.

Leverage-constrained agents. When some households are at the portfolio upper bound $a_R = a$, they are Leverage-constrained in the sense that they would like to tilt further toward the risky asset but cannot do so. In that case, the standard interior portfolio Euler equation no longer holds with equality at the individual level, and a shadow value of relaxing the constraint appears. Aggregating over these households generates the additional wedge χ, which captures how the risk-bearing capacity of leverage-constrained agents differs from that of unconstrained interior traders. Intuitively, interior households in U pin down the risk premium through their optimal portfolio choice, while leverage-constrained households in L are forced to hold an all-equity position regardless of whether that coincides with the unconstrained optimum.

The equity premium. Using the same consumption-weighted averages as in the expression for the risk-free rate, we can write the equity premium as the sum of two terms. To see this, consider the set of households U that hold both safe assets and risky equity. For each of these households, the portfolio first-order condition holds with equality. Taking the consumption-weighted average of this condition

over U gives

$$r_R - r_S = \underbrace{\gamma \sigma_R^2 \widetilde{E}^U \left[a_R(a,z) \frac{c_a(a,z)}{c(a,z)} \right]}_{\text{Consumption risk from equity}} + \underbrace{\psi_0 \widetilde{E}^U \left[\frac{a_R(a,z)}{a^{2+\psi_1}} \right]}_{\text{Marginal cost of holding equity}} . \tag{42}$$

The first term depends on how a change in the realized equity return affects household consumption. For instance, consider a household with wealth a and labor productivity z that experiences an unexpected one-percent loss on its equity holdings. Its wealth falls by $0.01 a_R(a,z)$. Since $c_a(a,z)$ is the marginal propensity to consume out of wealth, the corresponding change in consumption, as a fraction of its initial level, is approximately $-0.01 a_R(a,z) c_a(a,z)/c(a,z)$. Thus, the same equity loss has a larger effect on consumption for a household with a larger equity position, holding c_a/c fixed. The second term follows directly from the cost function in equation (6). At a given level of wealth a, shifting an additional dollar from the safe asset into equity raises the household's holding cost at the margin by $\psi_0 a_R(a,z)/a^{2+\psi_1}$.

We can also write the first term in equation (42) in terms of the covariance between consumption growth and the equity return. The equity shock dW_t^R enters the household's realized return with coefficient σ_R. The same shock enters household wealth with coefficient $\sigma_R a_R(a,z)$ and therefore enters consumption growth with coefficient $\sigma_R a_R(a,z) c_a(a,z)/c(a,z)$. Since the labor income shock is independent of the equity shock, the instantaneous covariance is given by

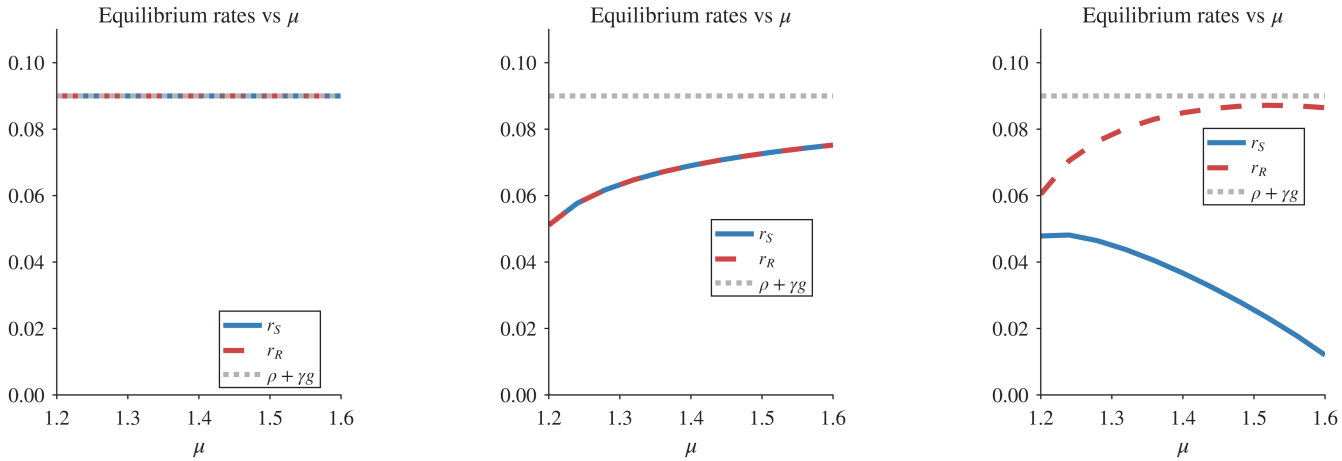
$$\frac{1}{dt} \operatorname{Cov}_t \left(\frac{dc_t}{c_t}, d\widetilde{r}_{R,t} \right) = \sigma_R \left(\sigma_R a_R(a,z) \frac{c_a(a,z)}{c(a,z)} \right) . \tag{43}$$

It follows that for each household with an interior portfolio,

$$r_R - r_S - \psi_0 \frac{a_R(a,z)}{a^{2+\psi_1}} = \gamma \frac{1}{dt} \operatorname{Cov}_t \left(\frac{dc_t}{c_t}, d\widetilde{r}_{R,t} \right), \qquad (a,z) \in U. \tag{44}$$

Equation (44) gives the familiar consumption-based interpretation of the equity premium.[20] A household requires compensation for holding equity when a low equity return coincides with a fall in its consumption, since this is precisely when the marginal utility of consumption is high. In our model, this covariance arises from uninsurable idiosyncratic equity risk. Equity shocks average out across households, so aggregate consumption has no exposure to these shocks. Yet each household's consumption responds to its own realized equity return. The first term in equation (42) therefore measures risk aversion times the consumption-weighted average of these household-level covariances.

[20] Constantinides and Duffie (1996) show that the equity premium can be positive even when aggregate consumption grows at a constant rate. In both their and our model, the premium depends on how equity returns covary with the marginal utility of household consumption. In our model, this covariance arises because a household's own equity return affects its consumption.



(a) Deterministic Ricardian economy

(b) One risk-free asset economy ($\sigma_R = 0, r = r_S = r_R$)

(c) Two-asset economy with safe debt and risky equity

Figure 2: Comparative statics of equilibrium interest rates with respect to markups μ. Panel (a) shows the deterministic Ricardian benchmark where $\sigma_z = \sigma_R = 0$. In Panel (b) $\sigma_R = 0$ so equity is risk-free and $r = r_S = r_R$. Panel (c) features both safe debt and risky equity ($\sigma_R > 0$), generating an endogenous spread $r_R - r_S$. All cases have $g = 0.02$, $\rho = 0.05$ and $\psi_0 = 0$ (no equity cost).

4 Limiting Cases and Comparative Statics

4.1 Limiting Cases

Full insurance. When agents can trade a full set of Arrow-Debreu securities, or idiosyncratic risk is shut down, consumption becomes a deterministic object, and it holds that:

$$\frac{\mathrm{d}c_t}{c_t} = \frac{1}{\gamma}(r_S - \rho - \gamma g) \tag{45}$$

That is, the risk-free rate is completely determined by **intertemporal substitution** motives and constant in the steady state $\rho + \gamma g = r_S$.

One asset. Now we focus on the standard incomplete markets model with just one risk-free asset. Following Kaplan et al. (2023), r_S can be written as:

$$r_S = \rho + \gamma g - \gamma \frac{\gamma + 1}{2} \widetilde{E}^U \left[\sigma_z(z)^2 \left(\frac{c_z}{c} \right)^2 \right] - \gamma \frac{C^C}{C^U} \widetilde{E}^C[\eta(z)] \tag{46}$$

which yields the standard incomplete-markets intuition: precautionary savings arising from idiosyncratic labor-income risk shift the risk-free rate below the deterministic balanced growth benchmark $\rho + \gamma g$.

Limiting cases. Figure 2 illustrates how equilibrium interest rates respond to changes in markups μ across three limiting cases of the model.

Panel 2a sets $\sigma_z = \sigma_R = \psi_0 = 0$, shutting down all idiosyncratic risk and costs related to holding assets. In this limiting case, households perfectly smooth consumption, and the precautionary motive vanishes. The stationary equilibrium condition reduces to the standard Euler equation, implying $r = r_S = r_R = \rho + \gamma g$. Because there is no risk and no binding borrowing-constraint distortion affecting intertemporal trade, markups affect only the levels of wages and profits, not the intertemporal price of consumption. As a result, equilibrium interest rates are invariant to μ.

Panel 2b sets $\sigma_R = \psi_0 = 0$, so equity becomes equivalent to the risk-free asset. In this case, $r = r_S = r_R$ and there is no equity premium. Although labor-income risk may still be present, the asset structure collapses to a single risk-free store of value.

An increase in markups raises monopoly profits and therefore increases the market value of claims to those profits. When these claims are risk-free, this expansion effectively increases the supply of safe stores of value in the economy. To clear the asset market, the equilibrium interest rate must rise. Consequently, both r_S and r_R increase with μ in this environment. This case corresponds to a fully capitalized profit channel operating through riskless wealth.

Panel 2c restores risky equity ($\sigma_R > 0$) but still without holding costs ($\psi_0 = 0$) and introduces a distinction between safe debt and risky equity. In this environment, r_S and r_R are determined separately in equilibrium. Two forces operate when markups increase.

First, markups reduce the marginal revenue product of capital through the factor $1/\mu$, which tends to reduce equilibrium capital. Since safe debt is constrained by collateral,$D \leq \lambda K$, a lower capital stock tightens the supply of safe debt. This contraction in the supply of safe assets puts downward pressure on the risk-free rate r_S.

Second, equity remains risky and must compensate households for bearing return risk. As the safe rate falls and portfolios rebalance, the required return on equity need not fall one-for-one. Instead, the equilibrium spread $r_R - r_S$ widens. In the figure, r_R increases modestly while r_S declines, generating a rising equity premium.

This limiting case illustrates the central mechanism of the model: markups alter the composition of asset supply. When safe debt is collateral-constrained and equity is risky, higher markups can simultaneously depress the risk-free rate and sustain or increase the required return on equity.

4.2 Comparative statics on markups.

An increase in markups μ generates a sharp decline in the safe rate r_S and a modest increase in the return on equity r_R, implying a widening equity premium $r_R - r_S$. At the same time, the labor share declines markedly while the profit share rises, reflecting the redistribution of income from labor toward profits. Welfare falls monotonically.

The central mechanism operates through the composition of household risk exposure. Higher markups shift household income and wealth away from labor income and toward equity claims on profits. This has two opposing effects. On the one hand, the lower labor share reduces households' exposure to idiosyncratic labor-income risk. On the other hand, the greater importance of equity in household wealth

raises exposure to equity-return risk. In our model, the second force dominates: households respond by increasing precautionary demand for safe assets, which bids up the price of safe claims and pushes down the equilibrium safe rate.

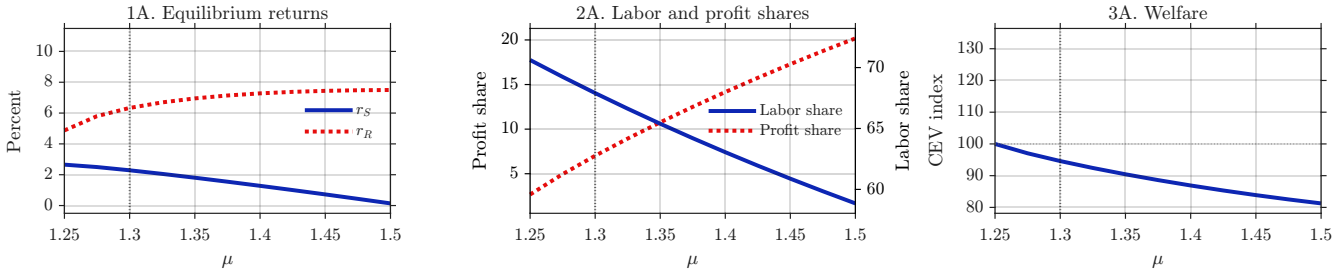


Figure 3: Comparative statics over markups μ

On the production side, higher markups also reduce the marginal revenue product of capital and depress capital accumulation. Since safe corporate debt is tied to collateral, lower capital reduces the supply of safe assets. However, this decline in capital is partially offset by the decline in the risk-free rate, which makes issuing debt cheaper and allows more capital to be financed. In general equilibrium, the supply of safe assets remains fairly constant due to these counteracting forces, while equity supply increases. Consequently, equity becomes a larger share of total assets in the economy. The rise in r_R reflects the increased importance of risky claims in aggregate wealth, together with the greater compensation required to bear equity risk. Welfare declines because markups generate deadweight loss, redistribute income toward risky profit income, increasing households' exposure to equity risk, and alter the distribution of wealth. The remainder of the comparative statics on other structural forces and how they shape the equilibrium returns and welfare are presented in Appendix B.

5 Quantification

In this section, we present the data sources used to construct moments and develop arguments for the identification of parameters used to match the moments. Thereafter, we fit the model to the data moments from 1984 until 2020. Finally, we show the fit of the model to some non-targeted moments including the risk-free rate and perform counterfactuals and welfare analysis.

5.1 Data

We summarize the data sources and definitions in Table 1 together with the externally calibrated parameters. Our goal is to match the model-generated moments with data. We collect data on US firms, households and macroeconomic aggregates over the period 1984-2020.

Firms. For the firm block, our primary source is Compustat, from which we retrieve annual data on total costs, total assets, and debt. We decompose total costs into administrative expenses (SGA), labor

costs, cost of goods sold, and capital costs. To produce markup estimates, we follow De Loecker et al. (2020). More specifically, each firm aggregate moment is computed as follows:

1. **Markups.** We aggregate markup estimates as: $\mu_t = \sum_i \mu_{it} \times cost_{it} / \sum_i cost_{it}$.
2. **Fixed costs.** For our measure of fixed to total costs we use the data to compute the ratio of total administrative expenses to total costs: $\Phi_t = \frac{\sum_i SGA_{it}}{\sum_i Tot.Cost_{it}}$.
3. **Debt to total assets.** In the model, we interpret Total Assets as Debt plus Internal Equity $K = D + E$. Since $D = \lambda K$, the collateral constraint can be matched by computing $\frac{\sum_i Debt_{it}}{\sum_i Tot.Assets_{it}}$.

Households. Our goal is to calibrate and validate different measures of household heterogeneity in our model. More specifically, we are interested in household inequality in labor earnings, wealth, and portfolio shares. For the household labor income process, we use the wage-dispersion series constructed by Heathcote et al. (2023) from the Current Population Survey. Our measure is the annual cross-sectional standard deviation of log hourly wages among men aged 25-60 who work at least 260 hours during the year. We obtain the standard deviation by taking the square root of the raw log-wage variance reported by Heathcote et al. (2023). Since we use raw wage dispersion, the series retains wage differences associated with observable worker characteristics. The measure captures inequality in hourly wages, which we interpret as a proxy for heterogeneity in labor productivity; it does not include variation arising from unemployment or annual hours worked. For wealth and portfolio shares of risky assets, we follow Xavier (2021) using the Survey of Consumer Finances (SCF). We consider risky assets as the sum of public and private equity: directly and indirectly held stocks—including equity held through mutual funds and retirement accounts—plus net equity in privately held businesses. Wealth is gross household assets, so the portfolio share is risky assets over total assets. Households are ranked by net worth when constructing percentiles of the wealth distribution.

Macroeconomic data. For the macroeconomic data, we retrieve from FRED (Federal Reserve Bank of St. Louis, 2026): the ten-year Treasury constant maturity rate and the profit share of total income. We use different deflators to transform nominal rates into real terms using: $r_t = (1 + i_t)/(1 + \pi_t) - 1$, where π_t denotes one of our yearly deflator measures. Our baseline deflator is the Bauer and Rudebusch (2020) Perceived Target Rate (PTR), which is the long-run inflation target perceived by market participants. The idea is to use a forward-looking variable instead of a backward-looking one to get closer to a true risk-free rate measure. We use CPI inflation from FRED as a robustness check. We obtain the equity premium data from Damodaran (2026), which constructs the expected return on equity using current prices and dividend forecasts from market analysts. Last, we match our labor-augmenting growth variable with Feenstra et al. (2015) TFP growth estimates for the US and divide them by the frictionless labor share $1 - \alpha$ to obtain the corresponding labor-augmenting series.

To remove excessive noise from the data and focus on structural trends, we take the five-year moving average (MA) of each series.

Identification. The six structural parameters are disciplined by six data moments. Cost-weighted markups and firms' debt-to-asset ratio discipline the markup parameter μ and the collateral parameter λ, respectively. Conditional on these parameters, the accounting profit share is informative about fixed costs ϕ. The standard deviation of log hourly wages disciplines labor-income dispersion ν, while TFP growth determines labor-augmenting productivity growth g. Finally, conditional on the calibrated coefficient of relative risk aversion γ, the equity premium is informative about equity-return risk σ_R. These links indicate which moments are particularly informative about each parameter, while the joint fit accounts for their interactions through general equilibrium. Appendix B.1 provides further details on identification.

Table 1: Calibration and estimation targets

Exogenously set parameters			
ρ	Discount rate	0.05	Achdou et al. (2021)
γ	CRRA parameter	2	Achdou et al. (2021)
θ_z	OU mean reversion	0.05	Storesletten et al. (2004)
α	Cobb–Douglas capital share	0.36	standard
δ	Depreciation	0.1	standard
ψ_0	Risky assets account cost	0.25	SCF portfolio shares
N	Number of firms per market	16	Eeckhout (2025)
θ	Between market elast. of subst.	1.2	De Loecker et al. (2021)
η	Within market elast. of subst.	5.75	De Loecker et al. (2021)
Calibrated / estimated parameters			
μ_t	Markups, cost weighted	$\widehat{\mu}_t$	Target: De Loecker et al. (2020)
ϕ_t	Fixed costs	$\widehat{\phi}_t$	Target: Acc. Profit Shares of Income
λ_t	Firms' collateral constraint	$\widehat{\lambda}_t$	Target: Debt to Total Assets ratio
$\sigma_{R,t}$	Risky-return volatility	$\widehat{\sigma}_{R,t}$	Target: Equity Premium Damodaran (2026)
ν_t	SD of log wages	$\widehat{\nu}_t$	Target: SD log wages
g_t	Labor-augmenting growth	$\widehat{g}_t$	Target: TFP growth Feenstra et al. (2015)

Notes: The estimated parameter paths $\{\widehat{\mu}_t, \widehat{\phi}_t, \widehat{\lambda}_t, \widehat{\sigma}_{R,t}, \widehat{\nu}_t\}$ are obtained by matching model-implied moments to data moments year-by-year using a Sobol grid of precomputed equilibria (nearest-neighbor criterion).

5.2 Moments and Estimates

Model fit. Figure 4 compares the time series implied by the estimated model with their empirical counterparts for the six targeted moments. The model is estimated by matching a five-year moving average of steady states period by period. We allow the structural parameters governing markups, fixed costs, collateral constraints, equity return risk, labor income risk and growth to vary so as to reproduce the joint evolution of the data moments.

Panel (a) reports cost-weighted markups. The model tracks both the upward trend and medium-run fluctuations in markups, capturing the persistent increase over the sample as well as the temporary plateau in the early 2000s before the sharp rise since the 2010s. The profit share of GDP, shown in panel (b), is also well reproduced. The model captures the rise during the 2000s, and the subsequent stabiliza-

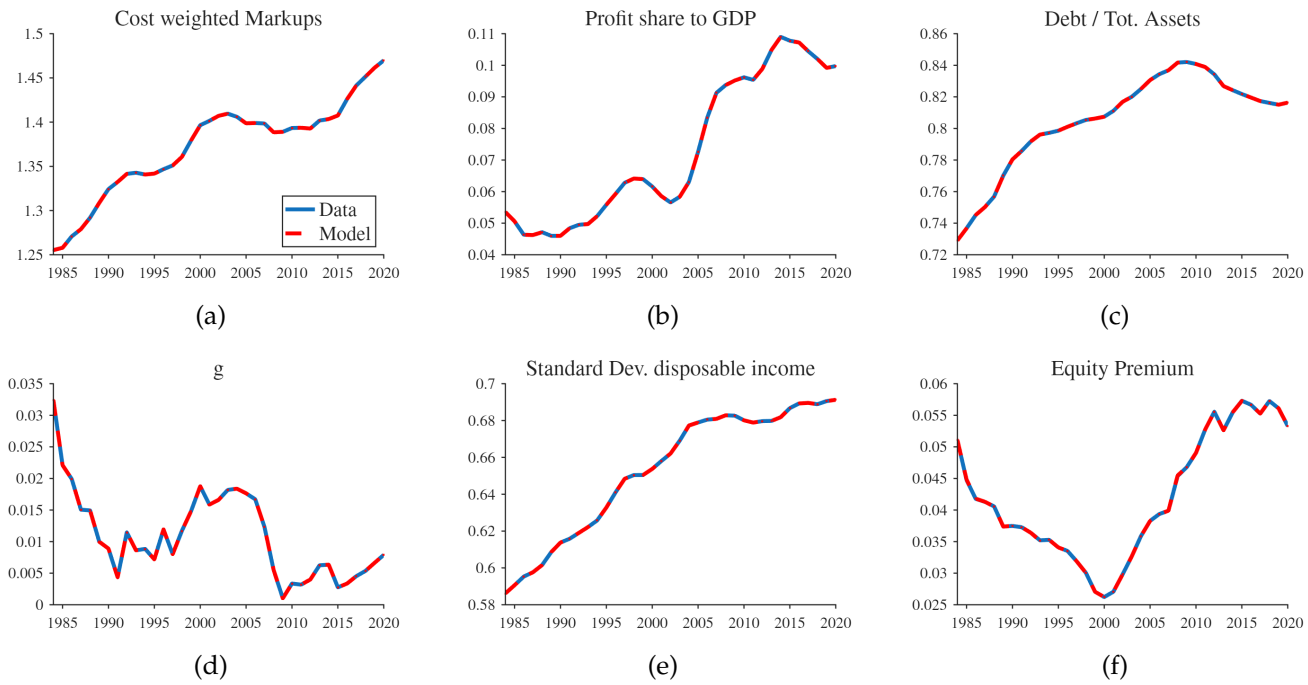


Figure 4: Model fit.

tion. This alignment suggests that the estimated fixed-cost parameter effectively accounts for changes in the share of firm profits in total income. It is key to match the profit share of income; otherwise we would be overestimating the implied stock market valuation and the effect of markups. Panel (c) compares the debt-to-asset ratio. The model replicates the gradual increase in leverage and its subsequent stabilization. The estimated collateral parameter therefore reproduces the increase in the debt-to-asset ratio in the data. Panel (d) shows the slowdown in productivity growth. The model perfectly tracks the long-run decline. Panel (e) shows the evolution of the standard deviation of disposable income between the data and the model, where the model matches the long-run increase in inequality perfectly. Panel (f) presents the equity premium, which steadily declines until the dot-com financial crash, then begins trending upward again. It stabilizes around 2015.

Overall, the model simultaneously matches the evolution of market power, profits, leverage, growth, inequality and the equity premium within a unified framework. The joint fit across real and financial moments indicates that the estimated structural parameters generate internally consistent movements in the model as in the data.

Estimated structural parameters. Figure 5 reports the time series of the estimated structural parameters. The markup parameter μ, shown in panel (a), exhibits a clear upward trend over the sample. It begins at a level of 1.26 in 1984 and rises toward 1.47 in 2020. The fixed cost parameter ϕ, reported in panel (b), increases from 0.28 in the early part of the sample to levels at 0.36 at its peak in the early 2000s, then declines around the mid-2000s and begins rising again toward the end of the sample. Overall, the estimates indicate a gradual increase in the importance of overhead costs similar to findings in De Ridder (2024). The collateral parameter λ, shown in panel (c), rises from 0.73 in early 1984 to 0.82 in 2020.

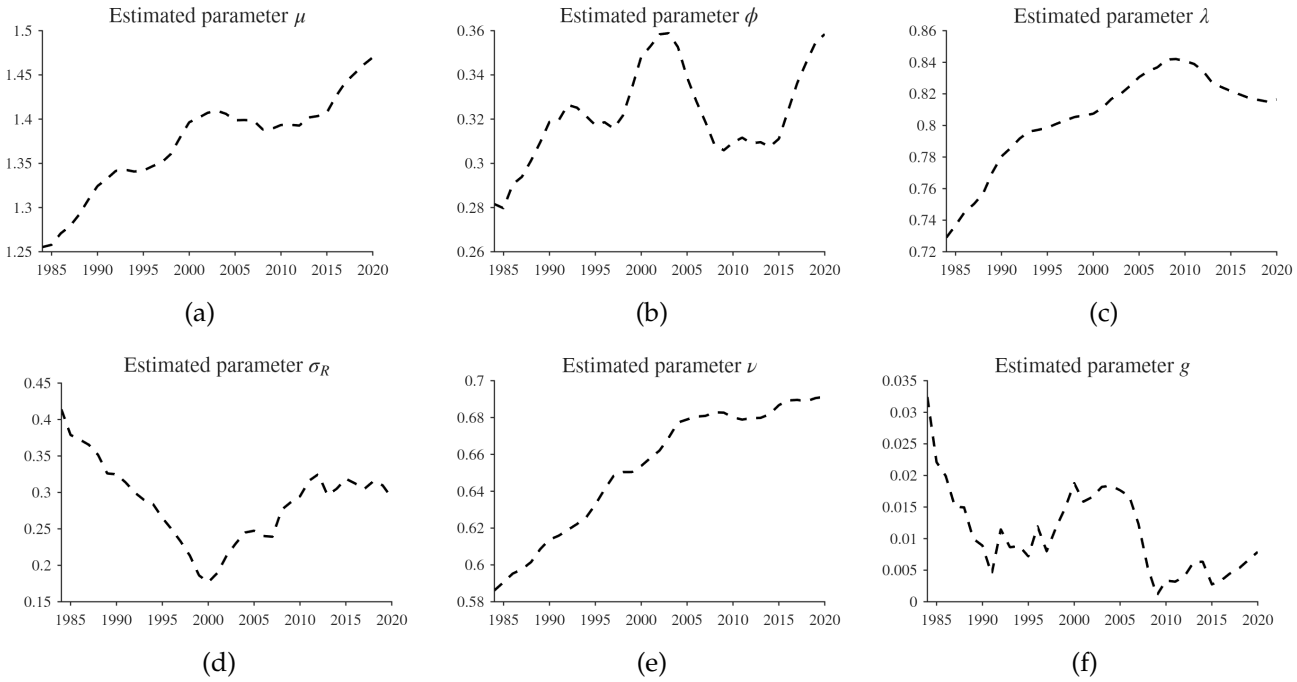


Figure 5: Estimated parameters.

The series is very smooth, suggesting a gradual loosening of borrowing constraints and consistent with higher leverage capacity over time. The volatility of equity returns σ_R, shown in panel (d), fluctuates from 0.41 in 1984 to 0.29 in 2020, with a dip in the early 2000s and a slight downward trend. The average estimate is 0.29, consistent with estimates of the cross-sectional standard deviation of risky returns in (Fagereng et al., 2016).[21] The stationary dispersion of log labor income ν, shown in panel (e), increases markedly over the sample. Its initial value is 0.59 and grows steadily up to 0.69 in 2020, reflecting a substantial increase in labor income risk. Finally, the productivity growth parameter shows a downward trend from 3.24% in 1984 to 0.79% in 2020, capturing the slowdown in US productivity growth over the last 36 years.

Taken together, the estimates suggest a structural environment characterized by rising market power, higher fixed operating costs, and greater labor income dispersion, accompanied by a gradual relaxation of collateral constraints, roughly stable equity return risk and a substantial slowdown in productivity growth.

Validation. Our model also captures several non-targeted structural trends which are plotted in Figure 6. Panel (a) shows the relative increase in the top 10% income share. Matching the change in income inequality is important to ensure we do not overshoot the decline in the risk-free rate from this channel.

[21] Our estimates of equity-return risk σ_R are conditional on the externally calibrated coefficient of relative risk aversion γ. This can be seen most clearly in the Merton portfolio rule, risk aversion and return variance enter through $\gamma\sigma_R^2$, so higher risk aversion implies lower return volatility for a given equity premium and risky portfolio share. Therefore, as an additional validation, Figure 7 compares the implied portfolio-return volatility, $\sigma_R a_R / a$, across wealth brackets with the estimates in Bach et al. (2020). The model broadly reproduces the increase in portfolio risk with wealth and generates comparable magnitudes across several brackets.

If inequality were to increase more in our model than in the data, households would be exposed to a much higher income risk, putting extra downward pressure on our risk-free rate estimates. In panel (B) we can see the evolution of the wealth share of the top 10%. Even though we do not match the initial change in the early 1980s, our model captures the overall trend. Panel (c) shows the evolution of the share of risky assets over total assets; while there is a level difference between the data and the model, the model closely approximates the overall increase since 1989. Similarly, in Panel (d) we see that the model fit for fixed costs over total costs, especially the shape of the change, implies that our profits and costs estimates are in line with overall trends in firms' cost composition. Panel (e) shows the equity in the model and the data, which is computed as in Farhi and Gourio (2018) [22]. The figure shows a remarkably close fit between the model and the data estimate throughout the sample. Finally, panel (f) shows the evolution of the labor share.

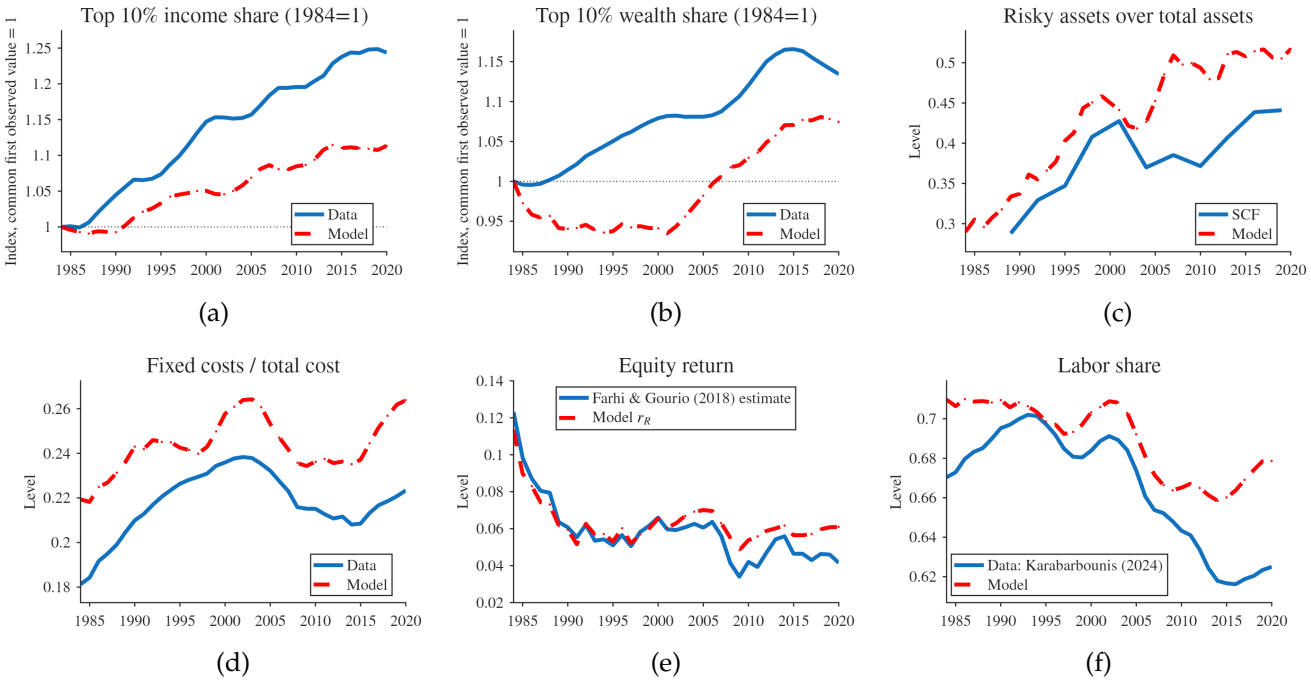


Figure 6: Non-targeted moments.

Idiosyncratic return risk validation. To provide external validation of our measure of idiosyncratic return risk (σ_R), we compare the household risk exposure implied by our estimated model with the empirical estimates in Bach et al. (2020). In our model, a household investing a fraction a_R/a of its financial wealth in equity faces annualized portfolio volatility $\sigma_R a_R/a$. Figure 7 averages this measure within wealth brackets and compares it with the average volatility of households' complete financial portfolios reported in their Table 5, column 5.[23] Their estimates use Swedish administrative data for

[22] Following Farhi and Gourio (2018), we use their data construction to obtain their plotted return series; however, contrary to their method, we do not take the 11-year moving average when building the input time series.

[23] The portfolio-return volatility reported in Bach et al. (2020) includes both systematic and idiosyncratic risk, whereas return risk in our model is entirely idiosyncratic. Asset coverage also differs between the two measures. We therefore interpret this comparison as an external check on the magnitude and wealth gradient of household portfolio risk. Despite these differences, the model provides a reasonably good fit

2000–2007. Our estimated model broadly reproduces the increase in portfolio risk with wealth, with the 2004 economy generating comparable magnitudes across several brackets. For instance, between the 60th and the 70th wealth percentiles, the annualized portfolio volatility is about 6% in both the model and the data. The model understates financial return volatility among poorer households relative to Bach et al. (2020). This may partly reflect the mass of households at the zero-wealth borrowing constraint in our model, which also implies zero equity holdings. In addition, among households with small positive wealth, equity holding costs further reduce risky portfolio shares and hence portfolio volatility. The 2020 economy generally implies greater exposure to return risk than the 2004 economy.

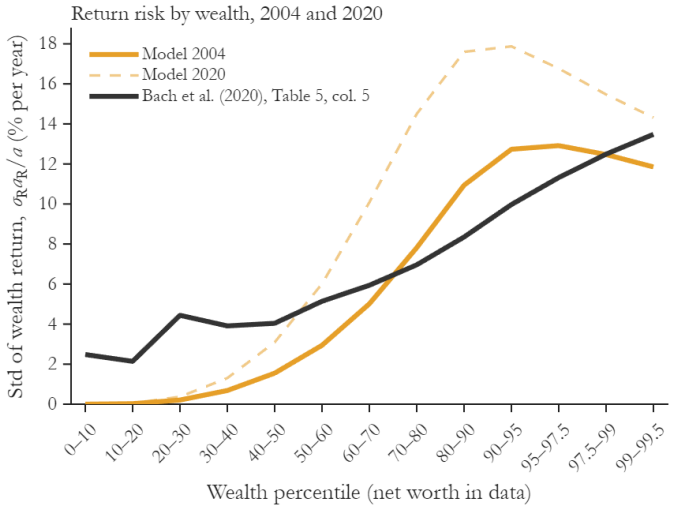


Figure 7: σ_R estimates validation.

Non-targeted risk-free rate. The risk-free rate is not targeted in our estimation. As a result, how well the risk-free rate in the model matches the data provides a useful test of the model. In Figure 8 we plot the model's risk-free rate in black along with several measures of the risk-free rate from the data. Our baseline is the 10-year Treasury rate using the perceived inflation target as in Bauer and Rudebusch (2020). In addition, we plot the 10-year Treasury rate deflated by CPI and the 1-year Treasury rate adjusted for inflation expectations as in Farhi and Gourio (2018). Although the model-implied risk-free rate and its counterparts in the data differ slightly in levels, the model replicates the main features of the data well: the relatively high risk-free rates of the 1980s, the increase around 2000, and the clear downward trend toward rates close to zero by 2020. Between 1984 and 2020, the decline across the three data measures ranges from 6.23 to 7.56 percentage points, while the model generates a decline of 5.43 percentage points. Given that the risk-free rate is entirely non-targeted, reproducing most of the observed decline provides an important validation of the model's mechanism.

In recent work, Impullitti and Rendahl (2025) show how the rise in market power can lead both to higher markups and lower growth, which affects wealth inequality as well as the risk-free interest rate. They do not have portfolio choice and the decline in the risk-free rate is entirely driven by the decline in productivity growth, not the capitalization of future firm profits. Our mechanism attributes the effect on the risk-free rate to declining markups as well as the decline in productivity growth, which through

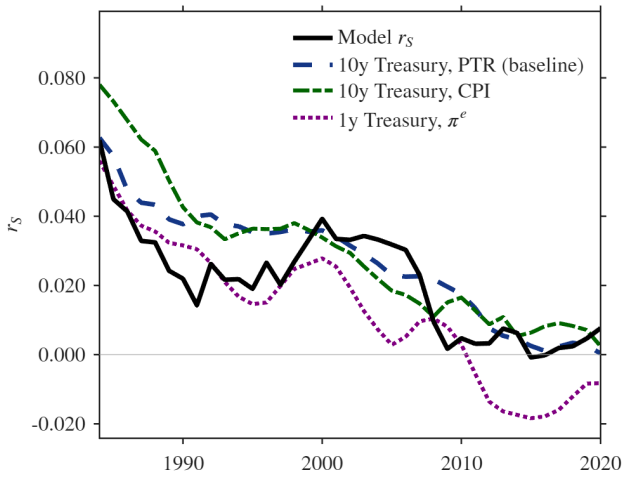


Figure 8: Model r_S and alternative measures.

Table 2: Risk-free rates, 1984–2020

	1984	2020	Δr_S
10y, PTR	6.27	0.03	−6.23
10y, CPI	7.80	0.24	−7.56
1y, π^e	5.60	−0.82	−6.43
Model	6.20	0.76	−5.43

Notes. Levels are in percent. $\Delta r_S = r_S^{2020} - r_S^{1984}$ is in percentage points. The data use a trailing five-year moving average within the 1984–2020 sample. PTR denotes the perceived inflation target; π^e denotes expected inflation.

the lens of the analysis in Impullitti and Rendahl (2025) are both due to rising market power. In other words, all of the decline in the risk-free rate, mediated through portfolio choice and growth, is due to the rise of market power.

5.3 Counterfactual of the Risk-free Rate

To assess the quantitative contribution of each structural force, we conduct a sequence of counterfactual experiments. Starting from a common initial steady state in 1984, we vary one estimated parameter at a time along its estimated path while holding all other parameters fixed at their baseline values.[24] By comparing these counterfactual paths, we quantify the relative importance of changes in markups, labor income risk, equity return volatility, the collateral constraint and growth for the decline in the risk-free rate. In the Appendix we also report the decomposition using the Shapley Value, for which we obtain quantitatively similar results.

Counterfactual decomposition of the risk-free rate. Figure 9 shows the counterfactual paths for the risk-free rate when we let one structural force at a time follow its estimated path, keeping the other parameters fixed at their 1984 values. Markups and the slowdown in productivity growth are the key drivers of the decline in the risk-free rate. If only markups μ change, the fall in r_S is close to the one in the baseline. The reason is that higher markups shift more income and wealth toward risky equity. As a result, the households want to hold more safe assets to hedge this additional exposure, which pushes down the safe rate. Fixed costs work in the opposite direction since they lower the profit share and therefore the equity risk exposure of the households. They reduce profits and therefore the amount of risky equity wealth. We use the term market-power to refer to the simultaneous change in markups μ

[24] We do not include the counterfactual change in fixed costs. The reason is that keeping markups at 1984 levels while increasing fixed costs implies negative profits and, in turn, negative stock market valuations. That is why we perform the counterfactuals with μ and ϕ together.

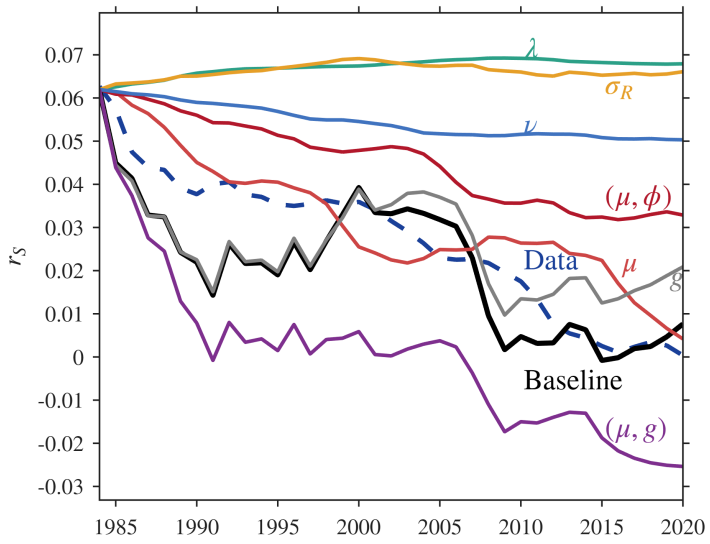


Figure 9: Risk-free-rate counterfactuals.

Table 3: Changes in the risk-free rate.

Δr_S	Counterfactual	Shapley
μ	−5.78	–
(μ, ϕ)	−2.91	−1.88
λ	0.60	0.62
σ_R	0.41	0.72
ν	−1.16	−0.95
g	−4.11	−3.94
Total (model)	−5.43	−5.43
Total (data)	−6.23	

Notes. $\Delta r_S = r_S^{2020} - r_S^{1984}$, in percentage points. Negative values indicate a decline. The Shapley decomposition treats (μ, ϕ) as one parameter. The model total reports the joint change when all parameters move, which equals the sum of the five Shapley contributions.

and fixed costs ϕ. Once μ and ϕ move together, the decline in r_S is much smaller, at roughly half of the total fall in the baseline.

The slowdown in productivity growth is the other important force. Letting only g move generates a fall in the safe rate of 4.11 percentage points. This works through the standard intertemporal channel in the model: a lower growth rate reduces the benchmark return $\rho + \gamma g$. The slowdown in growth therefore explains a large part of the decline in r_S, separately from the portfolio channel coming from market power.[25]

Labor-income risk ν also contributes to the decline of the risk-free rate. Its estimated change lowers the safe rate by 1.16 percentage points as higher uninsurable income risk strengthens precautionary saving. By contrast, the changes in λ and σ_R do not help explain the decline in r_S; along their estimated paths they actually move the safe rate slightly upward. The different counterfactuals should not be added mechanically, since the parameters interact in equilibrium. However, the figure makes clear that the main forces behind the decline of the risk-free rate are the slowdown in growth and the changes in product-market structure.

The counterfactual decomposition has two limitations. First, the model is highly non-linear, so attributing the underlying effect by only looking at the counterfactuals is an approximation. Second, we cannot produce a reliable counterfactual on the overhead cost parameter ϕ, as moving it to its 2020 value while keeping markups at their 1984 level implies negative profits. To overcome these limitations, Table 3 reports the counterfactual risk-free rates together with the Shapley values of each effect. We bundle markups μ and overhead costs ϕ to obtain realistic profit rates. Most models tie the evolution of markups and entry costs closely (see, for example, (De Loecker et al., 2021; De Ridder, 2024; Impullitti and Ren-

[25] As Auclert et al. (2025) point out, changes in productivity growth can have large effects on savings and interest rates when they change households' expected income growth, as in our model. Their baseline assigns productivity gains across cohorts, which weakens this effect.

dahl, 2025)), so their joint evolution can give us a more credible counterfactual of the change in market power. We find that markups and fixed costs together contribute 1.88 percentage points to the decline in the risk-free rate. The other values of the Shapley decomposition stay close to the counterfactual ones.

5.4 Mechanisms for the decline in the risk-free rate

To understand why markups play a central role in the decline of the risk-free rate, we revisit the key equation of our paper in equation (41) and analyze the mechanisms and tradeoffs behind the decline in the risk-free rate. In steady state the risk-free rate is

$$r_S = \underbrace{\rho + \gamma g}_{\text{BGP benchmark}} \underbrace{- \gamma \frac{\gamma+1}{2} \widetilde{E}^{U\cup L}\left[\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2\right]}_{\text{Precautionary: labor risk}} \underbrace{- \gamma \frac{\gamma+1}{2} \widetilde{E}^{U\cup L}\left[(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2\right]}_{\text{Precautionary: asset risk}} \underbrace{- \gamma \frac{C^C}{C^U + C^L} \widetilde{E}^{C}[\eta(z)]}_{\text{Hand-to-mouth}}$$
$$- \underbrace{\chi^{\Psi}}_{\text{Equity holding cost}} - \underbrace{\chi^{L}}_{\text{Leverage constraint}} . \tag{47}$$

Equation (47) decomposes the equilibrium risk-free rate into six components. The first term corresponds to the balanced-growth-path benchmark, $\rho + \gamma g$, which captures the standard effect of time preference and productivity growth on the intertemporal return. The second and the third terms are the wedges due to precautionary motives associated with labor-income risk and risky equity returns, respectively. The labor-risk term depends on households' exposure to idiosyncratic income shocks, while the asset-risk term depends on their exposure to risky equity through the portfolio position a_R and the sensitivity of consumption to wealth. The fourth term captures the consumption drift of hand-to-mouth households at the borrowing constraint. Finally, χ^{Ψ} captures the effect of the equity holding cost, while χ^{L} captures the shadow value of the leverage constraint. This decomposition allows us to identify which margins account for changes in the risk-free rate in the counterfactual exercises below.

Mechanism for markups: Figure (10) decomposes the markup counterfactual using the six terms in the equation (47). Since we hold productivity growth constant in this counterfactual, the BGP benchmark does not change. Therefore, as markups change the change in the risk-free rate comes entirely from the changes in the wedges generated by households' risk exposure and financial frictions.

As markups rise income shifts away from labor toward profits. This has the opposite effect on the two risk components. The labor-risk wedge therefore becomes smaller. Since this wedge enters negatively in equation (47), this force by itself would raise the risk-free rate. In the figure, the labor-risk component increases r_S by 0.74 percentage points over the sample.

The asset-risk channel works in the opposite direction. Higher markups increase the amount of wealth held in risky equity and therefore increase the households' exposure to equity-return risk. This raises the term

$$(\sigma_R a_R)^2 \left(\frac{c_a}{c}\right)^2,$$

strengthening precautionary demand for safe assets. The asset-risk component alone lowers the safe rate

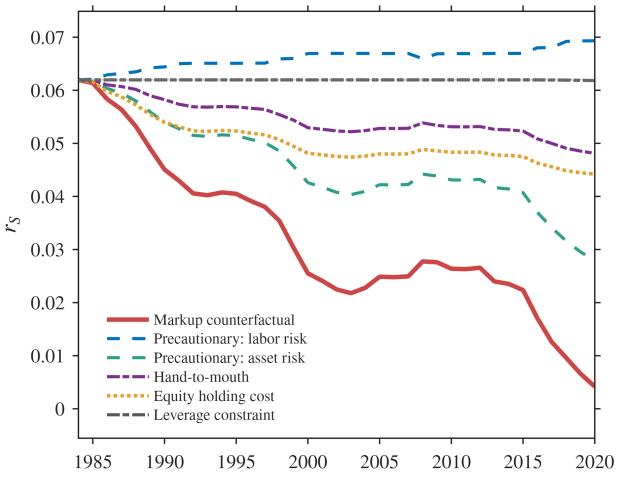


Figure 10: Markup risk-free rate counterfactual. Decomposition using equation (47).

by 3.41 percentage points and is the largest individual force behind the markup counterfactual.

The remaining wedges reinforce this decline. The hand-to-mouth component lowers the safe rate by 1.38 percentage points. The change in the equity-holding-cost term also contributes to a 1.78 percentage-point decline in r_S. By contrast, the leverage-constraint component is essentially zero over the counterfactual and plays almost no role in the response of the safe rate.[26]

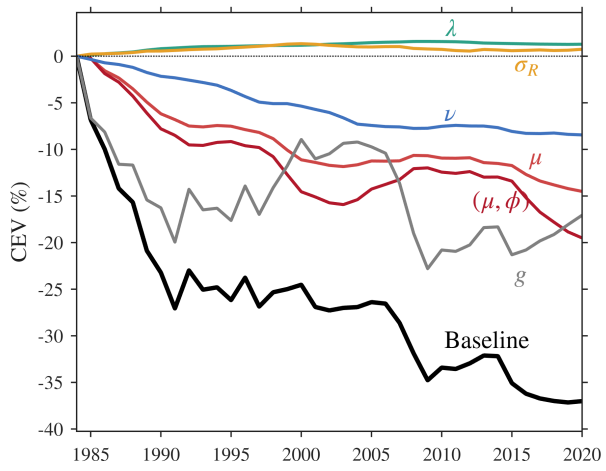
The decomposition therefore highlights the central tradeoff behind the effect of market power. Higher markups reduce exposure to labor-income risk, which in isolation would raise r_S. However at the same time they substantially increase exposure to risky equity, and this precautionary asset-risk channel more than offsets the decline in labor risk. The magnitude of this offsetting force, however, does depend on the amount of risk in equity σ_R^2. The higher is σ_R^2 relative to labor risk σ_z^2, the higher is the likelihood markups reduce the risk-free rate. This also explains why the result differs from models without an endogenous risk premium: in the absence of the asset-risk term, the decline in labor-risk exposure would instead push the risk-free rate upward, as in Auclert et al. (2025).

5.5 Counterfactuals on Welfare

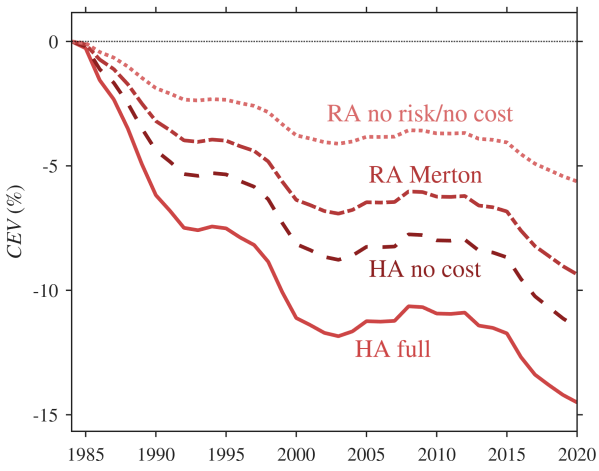
In this section, we use the measure in equation (105) to evaluate the welfare consequences of the structural changes since 1984. Figure 11 reports the welfare counterfactuals, expressed as consumption-equivalent variations relative to the 1984 economy.

Welfare counterfactual. Panel (a) shows the welfare consequences of varying each of the six structural forces separately, including one that bundles markups and fixed costs changing simultaneously. The baseline economy implies a substantial welfare loss of 37.01% by 2020. The decline in productivity growth generates a welfare loss of 17.06% , as slower growth lowers the path of consumption. The other

[26]The equity cost prevents households from hitting the leverage constraint as it disincentivizes positions with large equity portfolio shares.



(a) Consumption equivalent variation



(b) Decomposition of CEV counterfactuals over markups.

Figure 11: Welfare counterfactuals.

large contribution comes from a joint change in markups and fixed costs of 19.50%, whereas markups by themselves lead to a 14.51% decline in welfare. This welfare cost of markups is relatively high in comparison to conventional estimates of the costs of market power. In our model, changes in market power affect welfare not only through the standard production-side distortions associated with dead-weight loss and general equilibrium effects, but also by changing households' exposure to income and wealth risk. As markups increase, a larger share of income is paid out as profits, which are capitalized into risky equity, so households become more exposed to equity risk. This creates additional welfare losses that are absent when profit income is treated as riskless.

Understanding the markup effect on Welfare. Next, we explain why we find a larger welfare decline coming from markups. In panel (b), we zoom in on the markup counterfactual. Starting from our heterogeneous-agent economy (HA full), we shut down its ingredients one at a time and re-solve the general equilibrium of each simplified economy along the same markup path. Each line is measured relative to its own 1984 economy.

First, we remove the cost of holding equity ($\psi_0 = \psi_1 = 0$). The welfare loss in 2020 falls from 14.51% in HA full to 11.49% in HA no cost. Two forces account for this gap. First, the holding cost itself grows with markups: as profits are capitalized into a larger equity market, aggregate holding costs rise from 0.4% to 1.3% of GDP between 1984 and 2020, and these resources are lost to consumption. Second, the cost determines who holds the equity created by market power. Without it, wealth-poor households hold far more equity relative to their wealth than the rich. Rising markups shift income from wages, which every household earns, to profits. With the cost, those profits and the equity premium paid on them accrue mainly to the wealthiest households. Without it, poorer households share in both, which partly offsets their lower wages and mitigates the aggregate welfare loss.

Second, we shut down labor income risk ($\nu = 0$) and replace the borrowing and leverage constraints with the natural borrowing limit, while keeping equity-return risk. The model becomes a representative household that solves the consumption–portfolio problem of Merton (1969) and Angeletos (2007), in general equilibrium with our firm block (RA Merton).[27] The welfare loss falls to 9.36%.

Lastly, we shut down equity return risk, $\sigma_R = 0$, and the economy becomes a one-riskless-asset representative-agent economy, with a welfare change of -5.63% arising exclusively from the deadweight loss effect of markups. Importantly, our representative-agent economy implies an aggregate deadweight loss similar to that in De Loecker et al. (2021) and Edmond et al. (2023). Relative to them, we find the additional welfare losses are coming mostly through two new channels: risk exposure and the distribution of wealth. The comparison therefore helps isolate why the welfare effects are considerably larger in our baseline model. The decomposition shows that the conventional deadweight-loss channel accounts for only part of the welfare cost of rising markups: about 5.63 of the 14.51 percentage points. Adding risky equity raises the loss by 3.74 points. Household heterogeneity, labor-income risk and the borrowing constraint add the remaining 5.15 points when we match empirical portfolio heterogeneity.

5.6 Markup shocks, transitions and welfare

Although our analysis focuses on long-run structural forces, comparisons between stationary equilibria do not capture the welfare effects over the transition. In a representative agent economy for instance, higher markups can reduce investment and temporarily raise consumption, even when long-run consumption declines. These transitional consumption gains may therefore offset part of the welfare losses suggested by stationary comparisons. Moreover, Rachel (2025) shows that transitions may play an important role in the determination of the risk-free rate. In our model, adjustment also involves changes in asset valuations, wages, portfolio choices, and exposure to idiosyncratic risk. How these changes are distributed across the heterogeneous households shapes the evolution of aggregate consumption and welfare. We therefore examine whether the welfare implications of our stationary counterfactuals remain robust once we take transitional dynamics into account.[28]

We conduct two exercises, both starting from the 1984 stationary equilibrium. First, we examine the response to an unexpected, permanent 10% increase in markups, which is a change from $\mu^{1984} = 1.2553$ to $\mu^{+10\%} = 1.3808$ with perfect foresight thereafter, to identify the mechanisms governing the adjustment of consumption, risk exposure, and welfare. Second, we assume all agents learn the entire markup path as observed in the data from 1984 to 2019, and solve the resulting perfect-foresight transition. In the process, we hold all other structural parameters at their 1984 values. This exercise isolates the contribution

[27] Policies are linear in total wealth, financial wealth $D + V$ plus the present value of wages H, so each household holds a constant share $(r_R - r_S)/(\gamma\sigma_R^2)$ of total wealth in equity, and the economy features Gorman aggregation. Equity-market clearing sets this share equal to $V/(D + V + H)$, which pins down the premium. The safe rate follows from stationarity of aggregate consumption, the condition behind equation (41), in which only the equity-risk precautionary term remains. As in Angeletos (2007), return shocks are independent across a continuum of ex ante identical households.

[28] In this section, we focus on the effect of markups μ and keep the other 5 parameters $\{\phi, \lambda, \nu, \sigma_R, g\}$ constant. Re-estimating the whole parameter sequence under perfect foresight transitions requires additional assumptions about the information structure of the shocks, and is beyond the scope of the paper.

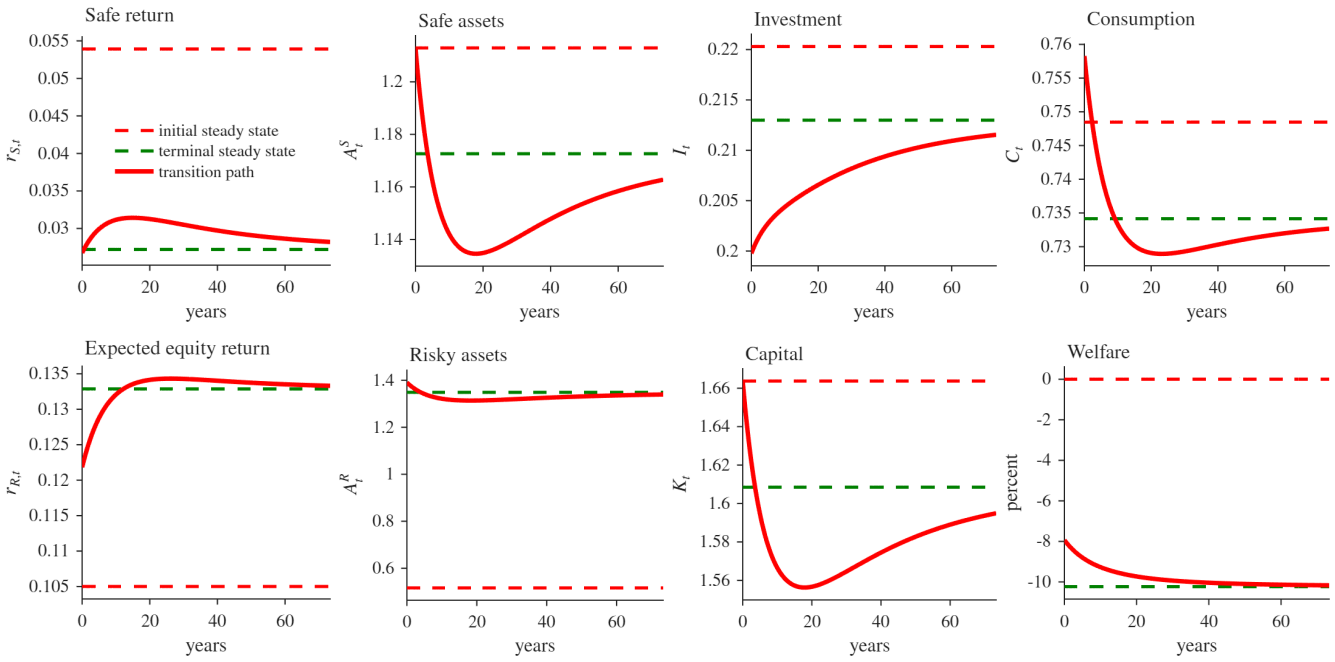


Figure 12: IRFs 10% μ shock

Note: dashed lines denote the initial steady-state value in red and the terminal steady-state value in green for each variable.

of rising markups and allows us to compare welfare accounting for the transition with the corresponding stationary counterfactual.

Impulse Response Functions. Figure 12 shows the response to an unexpected, permanent 1% increase in markups, with perfect foresight after the announcement. Higher markups affect the economy through two main channels: the distortion of production due to deadweight loss and the capitalization of monopoly profits into risky equity. We find that the safe rate (in red) on impact falls below its terminal steady state level (in green) before converging to it. Both supply and demand shape this transition. On impact, equity valuations jump and this increases the amount of wealth that the households hold in risky equity, which increases their exposure to idiosyncratic return risk, even though the volatility of return is unchanged. This increases the demand for safe assets and puts downward pressure on the risk-free rate. However, at the time of impact the supply of safe assets is tied to installed capital that cannot adjust until the next period. Lower investment leads to lower savings and higher consumption, which leads the risk-free rate to fall below its terminal steady state level. The expected equity return, in contrast, rises on impact and increases further during the transition. The equity premium therefore widens both initially and in the long run.

The supply of safe assets is predetermined on impact because firms borrow against installed capital; hence the asset supply changes only with gradual adjustments in capital. We find that the capital stock declines gradually, temporarily falling below its terminal level before recovering slightly. Its response reflects the interaction between the production distortion and changes in financing costs. Higher markups lower the marginal revenue product of capital, while the decline in the safe rate reduces the cost of debt finance and the increase in the expected equity return raises the cost of equity finance. The combined

effect in this exercise is a lower long-run capital stock. This also implies that investment falls because higher markups reduce the incentive to accumulate productive capital, and the changes in financing costs do not fully offset this effect. Therefore, as capital declines, corporate debt and aggregate safe-asset holdings fall proportionally.

For the risky assets, we observe a sharp jump at the point of impact. This is because the market value of equity jumps on impact as higher monopoly profits are capitalized into household wealth.

Consistent with the representative agent economy intuition, aggregate consumption initially increases as lower investment releases resources for consumption. However, the initial consumption gain is temporary and consumption subsequently falls below its initial level. Several forces are at play here; for instance for the households, the jump in equity valuations generates new wealth which puts upward pressure on consumption. At the same time, as capital declines the marginal product of labor falls, putting further downward pressure on real wages. The eventual consumption response depends on how these changes are distributed across households with different marginal propensities to consume. Consequently, the initial increase in aggregate consumption need not imply gains throughout the distribution as consumption subsequently falls below its initial level and converges to a lower steady state.

Despite the initial increase in aggregate consumption, we observe a large welfare loss on announcement; the consumption equivalent loss is approximately 7.93 percent on impact and approaches 10.23 percent in the terminal steady state. Welfare falls immediately because households anticipate the full consequences of permanently higher markups. Although lower investment and equity valuation gains temporarily support consumption, households also anticipate lower future wages and consumption and greater exposure to equity risk. These effects enter lifetime welfare at the announcement, before the capital stock has adjusted. The temporary increase in aggregate consumption is therefore insufficient to prevent a welfare loss at announcement. Most of the decline occurs at the announcement, with the remaining adjustment taking place over time, as the economy converges to the terminal steady state. We conclude that the welfare implications along the transition are similar to those along the comparison of steady states in the baseline quantitative exercise.

Markup Path Impulse shock. In Figure 13, we plot the impulse response functions after a shock in which the whole path of markups from 1984 to 2020 is announced, and from 2020 onward the markup level is kept at the 2020 level until the terminal steady state is reached. Most of the intuition from the 10% shock figure goes through, but there are some important differences. First, note that steady-state investment, capital, and safe assets increase rather than decrease. This is because of two counteracting effects, whose forces vary with the size of the permanent markup change. The markup increase directly lowers the optimal level of capital given some financial conditions, but in the long run, as interest rates fall, financing becomes cheaper and the optimal level of capital rises. Investment and safe assets adjust in line with that capital change.

A feature of the transition dynamics is the slow convergence towards the terminal steady state. By 2020, when markups stop changing, the risk-free rate is one percentage point from its steady-state value,

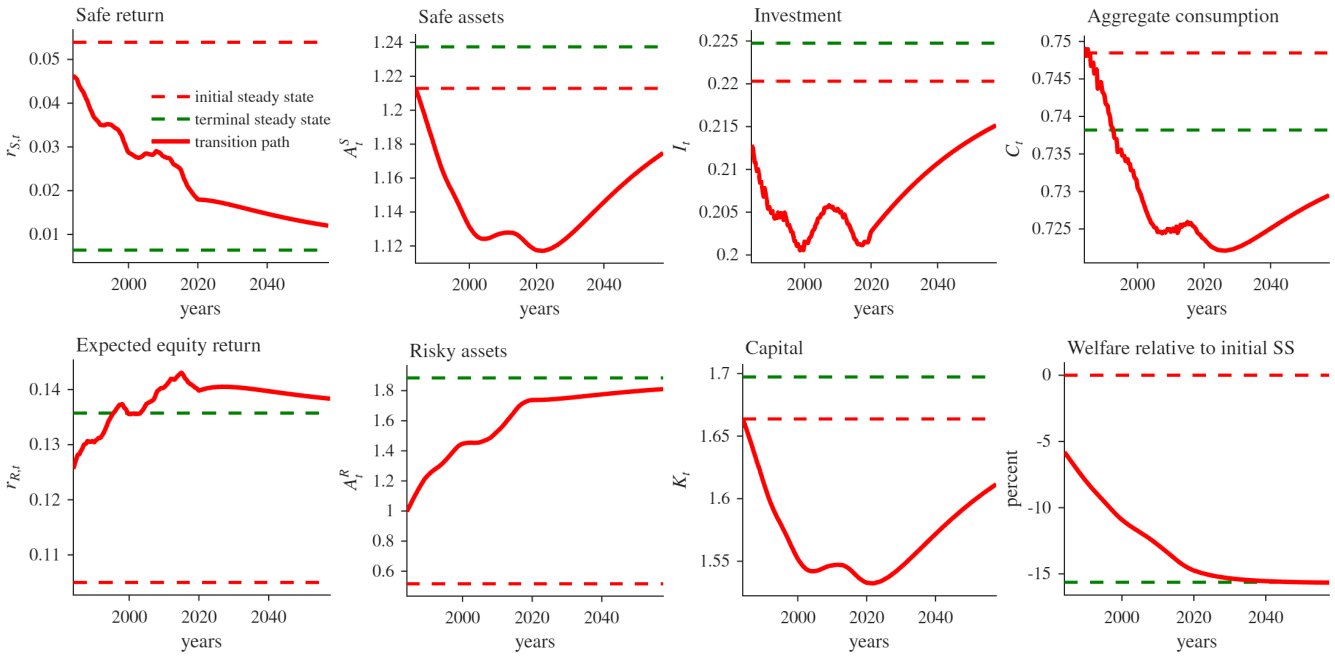


Figure 13: Full markup path

while capital and investment are very persistent. This is consistent with the mechanism described by Gabaix, Lasry, Lions, and Moll (2016). They show that the standard model with random growth generates very slow adjustments in the upper tail of the wealth distribution. Our idiosyncratic equity return risk generates the same random-growth mechanism, contributing to persistence in the wealth distribution. Gabaix et al. (2016) also identify two deviations from the random-growth model that can generate faster transitions: "scale dependence" and "type dependence". Scale dependence refers to individuals' income growth increasing with their income level, while type dependence separates individuals into different growth speeds, making some experience temporarily high income growth that can populate the tail faster. Our model features some scale dependence as equity holding costs make portfolio shares increase with wealth, but these costs fade with wealth and become negligible in the right tail of the distribution. Nevertheless, the purpose of this section is not to characterize the transition dynamics of the top of the wealth distribution but to validate our welfare results.

Welfare reaches its steady-state value much faster, by 2020. The tail of the distribution represents a small fraction of the population, and their contribution to aggregate welfare is limited by the concavity of the utility function. In addition, welfare is forward-looking and responds at the announcement to the anticipated path of consumption, wages, and returns. Hence, welfare losses are realized long before the economy reaches its terminal steady state.

6 Conclusion

In this paper we study whether the rise of market power can help explain the decline in the risk-free rate in the US since the early 1980s. Existing theories give different answers; some theories imply that

increasing markups reduce investment and asset supply and therefore lower the risk-free rate, while others show that once monopoly profits are capitalized into household wealth, higher markups can instead raise the risk-free rate. We build on the latter class of models and show that the risk attached to capitalized monopoly profits is central for determining whether higher markups increase or lower the risk-free rate.

To highlight the key role played by risky equity, we develop a heterogeneous agent model with safe corporate debt and risky equity claims on monopoly profits, where households face uninsurable labor and equity income risk. We derive a closed-form equilibrium characterization of the risk-free rate. We use this equation to highlight the two opposing forces due to higher markups. First, by shifting income away from labor, market power reduces households' exposure to labor income risk, which lowers the risk-free rate. At the same time, higher profits increase the amount of wealth held in risky equity, which raises the households' exposure to asset return risk and their demand for safe assets. When this second effect is sufficiently strong, higher market power lowers the risk-free rate. The main implication of our model is that the capitalization of profits is a key part of the story, but at the same time, the risk attached to the corresponding equity claims is crucial for how market power affects the equilibrium interest rates.

We quantify the model between 1984 and 2020 using data from the US. Despite not targeting the risk-free rate, the model reproduces much of its decline. Our counterfactual decomposition shows that rising markups can account for a sizable share of the decline in the risk-free rate, with the increase in households' exposure to risky equity playing a central role. The slowdown in productivity growth is also quantitatively important, while the remaining structural forces contribute only modestly. These results highlight the importance of the portfolio channel of markups in shaping the evolution of the risk-free rate.

The same mechanism has important welfare implications. In a representative-agent economy without risk, the welfare loss from rising markups is 5.63%, consistent with prior literature. Once we introduce risk in equity under a natural borrowing constraint the losses increase to 9.36% and household heterogeneity and incomplete insurance further increase the losses to almost 14.51%. Market power therefore affects welfare not only through distortions in production and investment due to deadweight loss, but also through the risk that households bear when monopoly profits are capitalized into equity.

More broadly, our paper shows that the interaction of rising markups with risky equity creates an important channel through which market power affects both the risk-free rate and welfare. As higher markups increase the value of risky equity claims, they change households' exposure to risk. This channel accounts for half of the decline in the risk-free rate and substantially amplifies the welfare cost of market power.

Supplementary Appendix

Appendix A Extra derivations

Household Side Derivations

In this subsection, we derive the household Euler/KKT conditions, the shadow values of the portfolio constraints, and the exact steady-state equation for the safe rate when households face a convex cost of holding risky equity. All household quantities are expressed in efficiency units, while r_S and r_R denote the level returns on the safe and risky assets.

Step 1: HJB. Let $V(a,z)$ be the stationary value function. The idiosyncratic income state follows

$$d\log z = -\theta\big(\log z - \log \bar{z}\big)\,dt + \sigma_z\,dW^z,$$

so that in levels

$$dz = \mu_z(z)\,dt + \sigma_z(z)\,dW^z, \qquad \mu_z(z) = \left[-\theta\big(\log z - \log \bar{z}\big) + \frac{1}{2}\sigma_z^2\right] z, \qquad \sigma_z(z)^2 = (\sigma_z z)^2.$$

Households choose consumption c and risky-asset holdings a_R subject to

$$a \geq \underline{a}, \qquad 0 \leq a_R \leq a - \underline{a}.$$

We allow for a convex holding cost of risky equity,

$$\Psi(a, a_R) \equiv \frac{\psi_0}{2}\frac{a_R^2}{a^{2+\psi_1}}. \tag{48}$$

The stationary HJB is

$$\begin{aligned}[\rho + (\gamma-1)g]\,V(a,z) = \max_{c\geq 0,\ a_R\in[0,a-\underline{a}]} \Big\{ & u(c) + V_a(a,z)\mu_a(a,z;a_R,c) + \tfrac{1}{2}V_{aa}(a,z)\,\sigma_a(a,z;a_R,c)^2 \\ & + V_z(a,z)\mu_z(z) + \tfrac{1}{2}V_{zz}(a,z)\,\sigma_z(z)^2\Big\},\end{aligned} \tag{49}$$

where

$$\begin{aligned}\mu_a(a,z;a_R,c) &= wz + (r_S - g)a + (r_R - r_S)a_R - c - \Psi(a,a_R),\\ \sigma_a(a,z;a_R,c)^2 &= (\sigma_R a_R)^2.\end{aligned}$$

In the quantitative exercises we set $\underline{a} = 0$, so the portfolio constraint becomes $0 \leq a_R \leq a$.

Step 2: KKT conditions (portfolio choice and state constraint). Households face three restrictions:

$$a \geq \underline{a}, \qquad a_R \geq 0, \qquad a_R \leq a - \underline{a}.$$

The borrowing limit $a \geq \underline{a}$ is a *state constraint*: in continuous time it is enforced through the boundary admissible set at $a = \underline{a}$, rather than through an interior pointwise KKT multiplier. It can be enforced by using the fact that the drift cannot be negative at the borrowing limit: $\partial_a V(\underline{a}, z) \geq u'(wz)$. Hence the static KKT multipliers below apply only to the portfolio bounds $a_R \geq 0$ and $a_R \leq a - \underline{a}$.

Let the multipliers for the portfolio constraints be

$$\zeta(a,z) \geq 0 \text{ for } (a - \underline{a} - a_R) \geq 0, \qquad \iota(a,z) \geq 0 \text{ for } a_R \geq 0.$$

(i) Consumption FOC.

$$u'(c) = V_a(a,z). \tag{50}$$

With CRRA utility, $u'(c) = c^{-\gamma}$, hence

$$V_a(a,z) = c(a,z)^{-\gamma} \qquad \Longleftrightarrow \qquad c(a,z) = V_a(a,z)^{-1/\gamma}.$$

(ii) Portfolio KKT condition. Since

$$\frac{\partial \Psi(a, a_R)}{\partial a_R} = \psi_0 \frac{a_R}{a^{2+\psi_1}},$$

differentiating the Lagrangian with respect to a_R gives

$$(r_R - r_S) V_a(a,z) - \psi_0 \frac{a_R(a,z)}{a^{2+\psi_1}} V_a(a,z) + \sigma_R^2 a_R(a,z) V_{aa}(a,z) - \zeta(a,z) + \iota(a,z) = 0. \tag{51}$$

(iii) Complementary slackness.

$$\zeta(a,z) \geq 0, \quad \zeta(a,z)\,(a - \underline{a} - a_R) = 0; \qquad \iota(a,z) \geq 0, \quad \iota(a,z)\, a_R = 0. \tag{52}$$

Region definitions. We partition the state space into four regions:

$$\begin{aligned}
U &= \{(a,z) : a > \underline{a},\ 0 < a_R(a,z) < a - \underline{a}\} && \text{(fully unconstrained off-boundary states)},\\
L^1 &= \{(a,z) : a > \underline{a},\ a_R(a,z) = a - \underline{a}\} && \text{(upper portfolio corner / leverage constrained)},\\
L^0 &= \{(a,z) : a > \underline{a},\ a_R(a,z) = 0\} && \text{(lower portfolio corner)},\\
C &= \{(a,z) : a = \underline{a} \mid \mathrm{d}a = 0\} && \text{(borrowing boundary / hand-to-mouth)}.
\end{aligned}$$

On U, one has $\zeta = \iota = 0$. On L^1, the upper portfolio bound binds, so $\zeta > 0$ and $\iota = 0$. On L^0, the lower portfolio bound binds, so $\zeta = 0$ and $\iota > 0$. On C, feasibility implies $a_R = 0$, and the borrowing limit is enforced through the state-constraint boundary condition.

Step 3: Premium equation and shadow values. Define absolute risk aversion

$$ARA(a,z) \equiv -\frac{V_{aa}(a,z)}{V_a(a,z)} = \gamma \frac{c_a(a,z)}{c(a,z)}.$$

Dividing (51) by $V_a(a,z)$ gives

$$r_R - r_S = \sigma_R^2 a_R(a,z)\, ARA(a,z) + \psi_0 \frac{a_R(a,z)}{a^{2+\psi_1}} + \frac{\zeta(a,z)}{V_a(a,z)} - \frac{\iota(a,z)}{V_a(a,z)}. \tag{53}$$

Interior premium equation on U. On U, $\zeta = \iota = 0$, so (53) reduces to

$$r_R - r_S = \sigma_R^2 a_R(a,z)\, \gamma \left(\frac{c_a(a,z)}{c(a,z)}\right) + \psi_0 \frac{a_R(a,z)}{a^{2+\psi_1}}, \qquad (a,z) \in U. \tag{54}$$

Equivalently, the interior risky position is

$$a_R(a,z) = -\frac{(r_R - r_S) V_a(a,z)}{\sigma_R^2 V_{aa}(a,z) - \psi_0\, V_a(a,z)/a^{2+\psi_1}}. \tag{55}$$

Upper-corner shadow value on L^1. On L^1, one has $\iota = 0$, so

$$\frac{\zeta(a,z)}{V_a(a,z)} = (r_R - r_S) - \sigma_R^2 a_R(a,z)\, \gamma \left(\frac{c_a(a,z)}{c(a,z)}\right) - \psi_0 \frac{a_R(a,z)}{a^{2+\psi_1}}, \qquad (a,z) \in L^1. \tag{56}$$

Lower-corner shadow value on L^0 has zero mass for a positive premium. On L^0, one has $\zeta = 0$ and $a_R = 0$, so (51) implies

$$\frac{\iota(a,z)}{V_a(a,z)} = -(r_R - r_S), \qquad (a,z) \in L^0. \tag{57}$$

Hence, if $r_R - r_S > 0$, an off-boundary lower-corner region L^0 is not KKT-consistent under this quadratic holding cost, because the marginal holding cost is zero at $a_R = 0$. Therefore, from here onwards, we let $L = L^1 \cup L^0$ denote the L^1-type agents since L^0 will have zero mass.

Step 4: Differentiate the HJB with respect to wealth a. The cost term depends on wealth through the denominator $a^{2+\psi_1}$. Its derivative is

$$\Psi_a(a, a_R) = -\frac{(2+\psi_1)\psi_0}{2} \frac{a_R^2}{a^{3+\psi_1}}.$$

It is convenient to define the positive object

$$\chi(a, a_R) \equiv -\Psi_a(a, a_R) = \frac{(2+\psi_1)\psi_0}{2} \frac{a_R^2}{a^{3+\psi_1}}. \tag{58}$$

Then differentiating the optimized HJB using the envelope theorem gives

$$[\rho + (\gamma - 1)g]\, V_a = V_{aa}\mu_a + \big(r_S - g + \chi(a, a_R)\big) V_a + \tfrac{1}{2} V_{aaa} \sigma_R^2 a_R^2 + V_{az}\mu_z(z) + \tfrac{1}{2} V_{azz} \sigma_z(z)^2 + \zeta(a,z),$$

or equivalently

$$(\rho + \gamma g - r_S - \chi(a, a_R))V_a = V_{aa}\mu_a + \tfrac{1}{2}V_{aaa}\sigma_R^2 a_R^2 + V_{az}\mu_z(z) + \tfrac{1}{2}V_{azz}\sigma_z(z)^2 + \zeta(a,z). \tag{59}$$

Note that, although $\iota(a,z)$ matters for portfolio choice on L^0, it does not appear in the a-derivative of the optimized HJB, because the lower portfolio bound $a_R \geq 0$ does not depend on wealth a. By contrast, the upper portfolio bound $a_R \leq a - \underline{a}$ does depend on a, and therefore contributes the term $\zeta(a,z)$.

Step 5: Apply Itô's lemma for $V_a(a,z)$ and for $c(a,z)$. Using

$$da = \mu_a(a,z;a_R,c)\,dt + \sigma_R a_R\,dW^R, \qquad dz = \mu_z(z)\,dt + \sigma_z(z)\,dW^z,$$

and $dW^R\,dW^z = 0$, Itô's lemma for $V_a(a,z)$ yields

$$dV_a = \Big((\rho + \gamma g - r_S - \chi(a,a_R))V_a - \zeta(a,z)\Big)\,dt + V_{aa}\sigma_R a_R\,dW^R + V_{az}\sigma_z(z)\,dW^z. \tag{60}$$

Now apply Itô's lemma to the stationary policy $c(a,z)$:

$$dc = c_a\,da + c_z\,dz + \tfrac{1}{2}c_{aa}(da)^2 + \tfrac{1}{2}c_{zz}(dz)^2.$$

Hence

$$dc = \Big[c_a\mu_a + \tfrac{1}{2}c_{aa}\sigma_R^2 a_R^2 + c_z\mu_z(z) + \tfrac{1}{2}c_{zz}\sigma_z(z)^2\Big]dt + c_a\sigma_R a_R\,dW^R + c_z\sigma_z(z)\,dW^z,$$

and therefore

$$(dc)^2 = \Big(c_a\sigma_R a_R\Big)^2 dt + \Big(c_z\sigma_z(z)\Big)^2 dt. \tag{61}$$

Since $V_a = c^{-\gamma}$, Itô's lemma gives

$$d(c^{-\gamma}) = -\gamma c^{-\gamma-1}dc + \tfrac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}(dc)^2.$$

But $d(c^{-\gamma}) = dV_a$. Taking conditional expectations and using (60) and (61), we obtain

$$\Big((\rho+\gamma g - r_S - \chi(a,a_R))c^{-\gamma} - \zeta(a,z)\Big)dt = -\gamma c^{-\gamma-1}E[dc \mid a,z] + \tfrac{1}{2}\gamma(\gamma+1)c^{-\gamma-2}\left[(c_a\sigma_R a_R)^2 + (c_z\sigma_z(z))^2\right]dt.$$

Solving for $E[dc \mid a,z]/dt$ gives

$$\begin{aligned}\frac{E[dc \mid a,z]}{dt} =& \frac{1}{\gamma}(r_S + \chi(a,a_R) - \rho - \gamma g)c(a,z) + \frac{1}{\gamma}\frac{\zeta(a,z)}{V_a(a,z)}\,c(a,z) \\ &+ \frac{\gamma+1}{2}\left[(\sigma_R a_R(a,z))^2\left(\frac{c_a(a,z)}{c(a,z)}\right)^2 + \sigma_z(z)^2\left(\frac{c_z(a,z)}{c(a,z)}\right)^2\right]c(a,z).\end{aligned} \tag{62}$$

Step 6: Aggregation. Let $m(a,z)$ denote the stationary density. Since consumption is expressed in efficiency units, stationarity refers to detrended aggregate consumption. Define consumption in each

region by

$$C^S \equiv \iint_S c(a,z)\, m(a,z)\, da\, dz, \qquad S \in \{U, L, C\},$$

For any region S, define the consumption-weighted average

$$\widetilde{E}^S[X] \equiv \frac{1}{C^S} \iint_S c(a,z)\, X(a,z)\, m(a,z)\, da\, dz.$$

Boundary region C. If $\underline{a} = 0$, then on the borrowing boundary one has $a_R = 0$ and $c = wz$, so

$$dc = w\, dz.$$

Defining

$$\eta(z) \equiv \frac{\mu_z(z)}{z},$$

we obtain

$$\frac{E[dc \mid a,z]}{dt} = c(a,z)\eta(z), \qquad (a,z) \in C. \tag{63}$$

Hence

$$\iint_C \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz = \iint_C c(a,z)\eta(z)\, m(a,z)\, da\, dz. \tag{64}$$

Upper-corner region L^1. On L^1, the upper portfolio bound binds and $\zeta > 0$, so

$$\begin{aligned}\iint_{L^1} \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz = {} & \frac{1}{\gamma}(r_S - \rho - \gamma g) C^{L^1} + \frac{1}{\gamma} \iint_{L^1} \chi(a, a_R) c(a,z) m(a,z)\, da\, dz \\ & + \frac{1}{\gamma} \iint_{L^1} \frac{\zeta(a,z)}{V_a(a,z)} c(a,z) m(a,z)\, da\, dz \\ & + \frac{\gamma+1}{2} \iint_{L^1} c(a,z) \left[\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R(a,z))^2 \left(\frac{c_a}{c}\right)^2 \right] m(a,z)\, da\, dz.\end{aligned} \tag{65}$$

Off-boundary states without an upper-corner shadow value: $U \cup L^0$. On both U and L^0, one has $\zeta(a,z) = 0$. Therefore

$$\begin{aligned}\iint_{U \cup L^0} \frac{E[dc \mid a,z]}{dt} m(a,z)\, da\, dz = {} & \frac{1}{\gamma}(r_S - \rho - \gamma g)\left(C^U + C^{L^0}\right) + \frac{1}{\gamma} \iint_{U \cup L^0} \chi(a, a_R) c(a,z) m(a,z)\, da\, dz \\ & + \frac{\gamma+1}{2} \iint_{U \cup L^0} c(a,z) \left[\sigma_z(z)^2 \left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R(a,z))^2 \left(\frac{c_a}{c}\right)^2 \right] m(a,z)\, da\, dz.\end{aligned} \tag{66}$$

Combining (64), (65), and (66), the stationarity condition $\mathrm{d}C/\mathrm{d}t = 0$ implies

$$0 = \frac{r_S - \rho - \gamma g}{\gamma}(C^U + C^L) + \frac{1}{\gamma}\iint_{U\cup L} \chi(a, a_R)c(a,z)m(a,z)\,da\,dz + \frac{1}{\gamma}\iint_{L} \frac{\zeta(a,z)}{V_a(a,z)}c(a,z)m(a,z)\,da\,dz$$
$$+ \frac{\gamma+1}{2}\iint_{U\cup L} c(a,z)\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R(a,z))^2\left(\frac{c_a}{c}\right)^2\right] m(a,z)\,da\,dz$$
$$+ \iint_{C} c(a,z)\eta(z)\,m(a,z)\,da\,dz. \tag{67}$$

Dividing by $(C^U + C^L)$ and using the consumption-weighted averages gives the exact safe-rate formula

$$r_S = \underbrace{\rho + \gamma g}_{\text{deterministic growth benchmark}} - \underbrace{\widetilde{E}^{U\cup L}[\chi(a,a_R)]}_{\text{wealth-relaxation effect of equity costs}} - \underbrace{\gamma\cdot\frac{\gamma+1}{2}\widetilde{E}^{U\cup L}\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right]}_{\text{precautionary saving pressure}}$$
$$- \underbrace{\frac{C^L}{(C^U+C^L)}\widetilde{E}^{L}\left[\frac{\zeta(a,z)}{V_a(a,z)}\right]}_{\text{upper-corner shadow value}} - \underbrace{\gamma\frac{C^C}{(C^U+C^L)}\widetilde{E}^{C}[\eta(z)]}_{\text{borrowing-boundary drift}}. \tag{68}$$

Step 7: Premium-gap representation. The interior Euler equality on U, (54), implies

$$r_R - r_S = \widetilde{E}^{U}\left[\sigma_R^2 a_R(a,z)\gamma\left(\frac{c_a(a,z)}{c(a,z)}\right) + \psi_0\frac{a_R(a,z)}{a^{2+\psi_1}}\right]. \tag{69}$$

Moreover, on $L^1 \equiv L$ one has $\iota = 0$, so (56) implies

$$\widetilde{E}^{L^1}\left[\frac{\zeta(a,z)}{V_a(a,z)}\right] = \widetilde{E}^{U}\left[\sigma_R^2 a_R(a,z)\gamma\left(\frac{c_a(a,z)}{c(a,z)}\right) + \psi_0\frac{a_R(a,z)}{a^{2+\psi_1}}\right] - \widetilde{E}^{L^1}\left[\sigma_R^2 a_R(a,z)\gamma\left(\frac{c_a(a,z)}{c(a,z)}\right) + \psi_0\frac{a_R(a,z)}{a^{2+\psi_1}}\right]. \tag{70}$$

Substituting (70) into (68) yields an equivalent premium-gap representation:

$$r_S = \underbrace{\rho + \gamma g}_{\text{deterministic growth benchmark}} - \underbrace{\widetilde{E}^{U\cup L}[\chi(a,a_R)]}_{\text{wealth-relaxation effect of equity costs}} - \underbrace{\gamma\cdot\frac{\gamma+1}{2}\widetilde{E}^{U\cup L}\left[\sigma_z(z)^2\left(\frac{c_z}{c}\right)^2 + (\sigma_R a_R)^2\left(\frac{c_a}{c}\right)^2\right]}_{\text{precautionary saving pressure}}$$
$$- \frac{C^L}{(C^U+C^L)}\underbrace{\left(\widetilde{E}^{U}\left[\sigma_R^2 a_R\,\gamma\left(\frac{c_a}{c}\right) + \psi_0\frac{a_R}{a^{2+\psi_1}}\right] - \widetilde{E}^{L}\left[\sigma_R^2 a_R\,\gamma\left(\frac{c_a}{c}\right) + \psi_0\frac{a_R}{a^{2+\psi_1}}\right]\right)}_{\text{how upper-corner households differ from marginal interior pricers}}$$
$$- \underbrace{\gamma\frac{C^C}{(C^U+C^L)}\widetilde{E}^{C}[\eta(z)]}_{\text{borrowing-boundary drift}}. \tag{71}$$

Solution Algorithm

We solve for the value function on a 2D grid in (a, z) using finite differences, following the standard continuous-time implicit method presented in Achdou et al. (2021). Starting from an initial guess V^0, iterate:

1. compute V_a with forward/backward differences and select the derivative by an *upwind scheme* (use the forward derivative where the implied drift $\mu_a > 0$ and the backward derivative where $\mu_a < 0$), while imposing a state constraint (no outflow) at $a = \underline{a}$ and a CRRA tail condition at $a_{\max}$;
2. update policies from the FOCs, $c^n = (V_a^n)^{-1/\gamma}$ and $a_R^{*,n} = -(r_R - r_S)V_a^n \left(\sigma_R^2 V_{aa}^n - \frac{\psi_0}{a^{2+\psi_1}} V_a^n\right)^{-1}$;;
3. with (c^n, k^n) fixed, assemble the discrete operator $A^n = A^{a,n} + A^z$, where $A^{a,n}$ uses upwinded drift and centered diffusion; the upwind construction ensures A^n is an *intensity matrix* (nonnegative off-diagonals and rows summing to zero);
4. update the value function using the *false-transient / implicit* step

$$\left(\tfrac{1}{\Delta} + \rho + (\gamma - 1)g\right)V^{n+1} - A^n V^{n+1} \;=\; u(c^n) + \tfrac{1}{\Delta}V^n,$$

   where $\Delta = 1000$, and stop when $\|V^{n+1} - V^n\|_\infty$ is below tolerance. It imitates VFI in a tiny step (continuous time).

Firm's Hamiltonian Full derivation

Recall the firm's problem is to maximize the present discounted stream of dividends subject to capital and debt accumulation constraints plus a collateral constraint. To do so, it chooses labor L_t, investment I_t and borrowing B_t. Since firm quantities are expressed in efficiency units, the discount rate of detrended dividends is $r_R - g$, while capital and debt evolve according to $\dot{K}_t = I_t - (\delta + g)K_t$ and $\dot{D}_t = B_t - gD_t$. This translates to the following current-value Hamiltonian:

$$\mathcal{H}_t = \Pi_t^o - w_t\phi - I_t - r_S D_t + B_t + q_t\left(I_t - (\delta + g)K_t\right) + m_t^D\left(B_t - gD_t\right) + \xi_t\left(\lambda K_t - D_t\right). \tag{72}$$

where, recall $\Pi_t^o = p_t Y_t - w_t L_t^P$. The first-order conditions are thus:

$$\frac{\partial \mathcal{H}_t}{\partial L_t} = \frac{\partial \Pi_t^o}{\partial L_t^P} = 0, \tag{73}$$

$$\frac{\partial \mathcal{H}_t}{\partial I_t} = -1 + q_t = 0, \tag{74}$$

$$\frac{\partial \mathcal{H}_t}{\partial B_t} = 1 + m_t^D = 0. \tag{75}$$

The investment and borrowing conditions imply

$$q_t = 1, \qquad m_t^D = -1. \tag{76}$$

Using the expression for operating profits, the labor first-order condition can be written as

$$\frac{\partial(p_t Y_t)}{\partial L_t^P} = w_t. \tag{77}$$

Under our market structure, marginal revenue equals price divided by the gross markup μ, so this becomes

$$\frac{p_t}{\mu}(1-\alpha)K_t^{\alpha} L_t^{-\alpha} = w_t. \tag{78}$$

The costate equation for installed capital is

$$dq_t = \left[(r_R - g)q_t - \frac{\partial \mathcal{H}_t}{\partial K_t}\right] dt. \tag{79}$$

Since

$$\frac{\partial \mathcal{H}_t}{\partial K_t} = \frac{\partial \Pi_t^o}{\partial K_t} - (\delta + g)q_t + \lambda \xi_t, \tag{80}$$

we obtain

$$dq_t = \left[(r_R + \delta)q_t - \frac{\partial \Pi_t^o}{\partial K_t} - \lambda \xi_t\right] dt. \tag{81}$$

Using the production function, this becomes

$$dq_t = \left[(r_R + \delta)q_t - \frac{p_t}{\mu}\alpha A K_t^{\alpha-1} L_t^{P1-\alpha} - \lambda \xi_t\right] dt. \tag{82}$$

The costate equation for debt is

$$dm_t^D = \left[(r_R - g)m_t^D - \frac{\partial \mathcal{H}_t}{\partial D_t}\right] dt. \tag{83}$$

Since

$$\frac{\partial \mathcal{H}_t}{\partial D_t} = -r_S - g m_t^D - \xi_t, \tag{84}$$

we obtain

$$dm_t^D = \left[r_R m_t^D + r_S + \xi_t\right] dt. \tag{85}$$

In steady state, $dq_t = 0$ and $dm_t^D = 0$. Using $q_t = 1$ and $m_t^D = -1$, the costate equations imply

$$r_R + \delta = \frac{p_t}{\mu}\alpha A K_t^{\alpha-1} L_t^{P1-\alpha} + \lambda \xi_t, \tag{86}$$

$$\xi_t = r_R - r_S. \tag{87}$$

Hence, for a positive premium, $\xi_t > 0$ and the collateral constraint binds, $D_t = \lambda K_t$. The steady-state capital condition can be written as

$$\lambda r_S + (1-\lambda) r_R + \delta = \frac{p_t}{\mu}\alpha A K_t^{\alpha-1} L_t^{P1-\alpha}. \tag{88}$$

Common Ownership: Derivation of the Markup

We follow the derivation in Eeckhout (2025). The final goods producer aggregates the differentiated intermediate inputs produced in the oligopolistically competitive market. The final goods producer chooses inputs Y_{njt} to maximize profits. As in Jaimovich and Floetotto (2008) and Atkeson and Burstein (2008), optimization by the final goods producers leads to log-linear demand and inverse demand functions for each input good Y_{njt}:

$$Y_{njt} = \frac{1}{N}\left(\frac{p_{njt}}{p_{jt}}\right)^{-\eta}\left(\frac{p_{jt}}{p_t}\right)^{-\theta} Y_t \quad \text{and} \quad p_{njt} = N^{-\frac{1}{\eta}}\left(\frac{Y_{njt}}{Y_{jt}}\right)^{-\frac{1}{\eta}}\left(\frac{Y_{jt}}{Y_t}\right)^{-\frac{1}{\theta}} p_t. \tag{89}$$

where p_{njt} is the price of input good nj, p_{jt} is the market price index, and p is the economy-wide aggregate price index.

We first solve the static optimization problem to derive the residual demand elasticity and the markup. Each firm holds a share κ of the other $N-1$ firms, where $\kappa \in [0, 1/N]$. The objective of the owner of firm n is therefore

$$\text{Max}_{Y_{nj}} \quad \tilde{\pi}_{nj} = (1-(N-1)\kappa)\pi_{nj} + \kappa \sum_{m \neq n} \pi_{mj} \tag{90}$$

Let the user cost of capital be

$$u_t = \lambda r_{S,t} + (1-\lambda) r_{R,t} + \delta$$

Then the first-order condition with respect to Y_{nj} is

$$(1-(N-1)\kappa)\left(p_{nj} + Y_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}} - w_t \frac{\partial L_{nj}}{\partial Y_{nj}} - u_t \frac{\partial K_{nj}}{\partial Y_{nj}}\right) + \kappa \sum_{m \neq n} Y_{mj}\frac{\partial p_{mj}}{\partial Y_{nj}} = 0 \tag{91}$$

$$(1-(N-1)\kappa)\left(p_{nj} + Y_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\right) + \kappa \sum_{m \neq n} Y_{mj}\frac{\partial p_{mj}}{\partial Y_{nj}} = (1-(N-1)\kappa)\left(w_t \frac{\partial L_{nj}}{\partial Y_{nj}} + u_t \frac{\partial K_{nj}}{\partial Y_{nj}}\right) \tag{92}$$

$$\left(p_{nj} + Y_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\right) + \underbrace{\frac{\kappa}{(1-(N-1)\kappa)}}_{=\Omega_t:\text{Relative ownership of other firms}} \sum_{m \neq n} Y_{mj}\frac{\partial p_{mj}}{\partial Y_{nj}} = w_t \frac{\partial L_{nj}}{\partial Y_{nj}} + u_t \frac{\partial K_{nj}}{\partial Y_{nj}} \tag{93}$$

$$\left(p_{nj} + Y_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\right) + \Omega_t \sum_{m \neq n} Y_{mj}\frac{\partial p_{mj}}{\partial Y_{nj}} = w_t \frac{\partial L_{nj}}{\partial Y_{nj}} + u_t \frac{\partial K_{nj}}{\partial Y_{nj}} \tag{94}$$

Now substitute the own and cross-price elasticities

$$\left(p_{nj}+p_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{nj}}\right)+\Omega_t\sum_{m\neq n}Y_{mj}\frac{\partial p_{mj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{mj}}\frac{p_{mj}}{Y_{nj}}\frac{p_{nj}}{p_{nj}}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{95}$$

$$\left(p_{nj}+p_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{nj}}\right)+\Omega_t\sum_{m\neq n}\frac{\partial p_{mj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{mj}}\frac{Y_{mj}p_{mj}}{Y_{nj}p_{nj}}p_{nj}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{96}$$

$$\left(p_{nj}+p_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{nj}}\right)+\Omega_t\sum_{m\neq n}\frac{\partial p_{mj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{mj}}\frac{Y_{mj}p_{mj}/\left(\sum_i^N p_{nj}Y_{nj}\right)}{Y_{nj}p_{nj}/\left(\sum_i^N p_{nj}Y_{nj}\right)}p_{nj}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{97}$$

$$\left(p_{nj}+p_{nj}\frac{\partial p_{nj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{nj}}\right)+\Omega_t\sum_{m\neq n}\frac{\partial p_{mj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{mj}}\frac{s_{mj}}{s_{nj}}p_{nj}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}}, \tag{98}$$

where s_{nj} is the revenue share of firm n in its market j. Given the nested CES demand system, it follows that $\frac{\partial p_{mj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{mj}}=\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}$ and $\frac{\partial p_{nj}}{\partial Y_{nj}}\frac{Y_{nj}}{p_{nj}}=-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}$ for all n. Then write the above equation as:

$$\left(p_{nj}+p_{nj}\left[-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}\right]\right)+\Omega_t\sum_{m\neq n}\left[\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}\frac{s_{mj}}{s_{nj}}\right]p_{nj}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{99}$$

or

$$\left(p_{nj}+p_{nj}\left[-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}\right]\right)+\Omega_t\sum_{m\neq n}\left[\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{mj}\right]p_{nj}=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{100}$$

$$p_{nj}\left(1-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{nj}+\Omega_t\sum_{m\neq n}\left(\frac{1}{\eta}-\frac{1}{\theta}\right)s_{mj}\right)=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{101}$$

$$p_{nj}\left[1-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)\left(s_{nj}+\Omega_t\sum_{m\neq n}s_{mj}\right)\right]=w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}} \tag{102}$$

$$\left[1-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)\left(s_{nj}+\Omega_t\sum_{m\neq n}s_{mj}\right)\right]^{-1}=\frac{p_{nj}}{w_t\frac{\partial L_{nj}}{\partial Y_{nj}}+u_t\frac{\partial K_{nj}}{\partial Y_{nj}}} \tag{103}$$

where the right-hand side is the firm's markup.

With identical firms, $s_{nj}=\frac{1}{N}$, and therefore the markup, common to all firms, is $\mu_{njt}=\mu_t$:

$$\mu_t=\left[1-\frac{1}{\eta}+\left(\frac{1}{\eta}-\frac{1}{\theta}\right)\frac{1}{N}\left(1+\Omega_t(N-1)\right)\right]^{-1} \tag{104}$$

Welfare

We compare welfare across stationary equilibria using a consumption-equivalent variation. Consider a benchmark steady state A and a counterfactual steady state B. In this environment, welfare depends not only on average consumption, but also on idiosyncratic labor-income risk, risky-asset exposure, self-insurance, portfolio constraints, and the equilibrium prices induced by each steady state. For this reason,

the natural welfare object is the household value function itself.

Let $V^A(a,z)$ and $V^B(a,z)$ denote the stationary value functions in the benchmark and counterfactual steady states, respectively, for a household with assets a and labor productivity z. With the CRRA utility, the state-by-state consumption-equivalent variation, denoted $CEV(a,z)$, is defined by

$$V^B(a,z) = (1 + CEV(a,z))^{1-\gamma} V^A(a,z).$$

Hence,

$$CEV(a,z) = \left(\frac{V^B(a,z)}{V^A(a,z)}\right)^{\frac{1}{1-\gamma}} - 1.$$

This object gives the proportional change in consumption in the benchmark steady state that would make a household in state (a,z) indifferent between the benchmark and the counterfactual. To summarize welfare at the aggregate level, we average each economy's value function over its own stationary distribution. Define

$$\bar{V}^A = \int V^A(a,z) m^A(a,z)\, da\, dz, \qquad \bar{V}^B = \int V^B(a,z) m^B(a,z)\, da\, dz.$$

$\bar{V}^j$ is the expected lifetime utility of a household drawn from the stationary distribution of economy j, so $\overline{CEV}$ compares the populations of the two stationary economies. It does not account for transitions between them, but we address transitions in Section (5.6). The aggregate consumption-equivalent variation $\overline{CEV}$ is then defined by

$$\bar{V}^B = \left(1 + \overline{CEV}\right)^{1-\gamma} \bar{V}^A,$$

or equivalently,

$$\overline{CEV} = \left(\frac{\bar{V}^B}{\bar{V}^A}\right)^{\frac{1}{1-\gamma}} - 1. \tag{105}$$

This welfare comparison is useful because it preserves the full structure of the household problem. It incorporates changes in consumption, labor-income risk, risky-asset risk, borrowing constraints, portfolio constraints, and equilibrium prices within a single measure. Throughout the numerical exercises, the baseline year for welfare calculations will be 1984.

Transitional dynamics theory and computation

In this subsection, we describe the model outside steady state and the shocks, and how we compute the perfect-foresight transition dynamics after a markup MIT shock is announced. That is, at $t = 0$, which we interpret as our starting year (1984), there is an unanticipated announcement about the entire future path of markups $\{\mu_t\}_{t\geq 0}$. The only new ingredient is that aggregate prices and quantities are deterministic functions of calendar time. All other parameters are held at their 1984 values, excepting markups.

Households. With time-varying prices, the HJB acquires a time derivative to be solved backward in time:

$$[\rho + (\gamma - 1)g]\ V_t(a,z) = \partial_t V_t(a,z) + \max_{c\geq 0,\ a_R\in[0,a]} \Big\{ u(c) + \partial_a V_t\, \mu_{a,t} + \tfrac{1}{2}\partial_{aa} V_t\, \sigma_R^2 a_R^2 + \mu_z(z)\partial_z V_t + \tfrac{1}{2}\sigma_z(z)^2 \partial_{zz} V_t \Big\},$$

where

$$\mu_{a,t} = w_t z + (r_{S,t} - g)a + (r_{R,t} - r_{S,t})a_R - \Psi(a, a_R) - c.$$

The new elements are the term $\partial_t V_t$ and the date-t prices $(w_t, r_{S,t}, r_{R,t})$ in the drift. For example, the state constraint at the borrowing limit is now $\partial_a V_t(0,z) \geq u'(w_t z)$.

We will solve the optimization problem backward. Thus, we need to impose a terminal condition, which corresponds to the steady-state value function associated with the new set of parameters. Prices converge to their terminal values only asymptotically, because the distribution does, so at a long horizon T we impose the terminal stationary value function, $V_T = V^{20}$.

Where V_t is concave in wealth, the first-order conditions are static in $(\partial_a V_t, \partial_{aa} V_t)$ and date-t prices, so they take the same form as before, date by date:

$$\sigma_R\, a^*_{R,t} = \frac{r_{R,t} - r_{S,t}}{\sigma_R}\Big(\frac{-\partial_{aa} V_t}{\partial_a V_t}\Big)^{-1}\Big[1 + \frac{\psi_0}{\sigma_R^2\, a^{2+\psi_1}}\Big(\frac{-\partial_a V_t}{\partial_{aa} V_t}\Big)\Big]^{-1},$$

$$c_t = (\partial_a V_t)^{-1/\gamma}, \qquad a_{R,t} = \min\big\{\max\{a^*_{R,t}, 0\},\, a - \underline{a}\big\}. \tag{106}$$

Distribution. The density $m_t(a,z)$ evolves according to the time-dependent Kolmogorov forward equation

$$\partial_t m_t = -\partial_a\big[\mu_{a,t} m_t\big] + \tfrac{1}{2}\partial_{aa}\big[\sigma_R^2 a_{R,t}^2 m_t\big] - \partial_z\big[\mu_z(z) m_t\big] + \tfrac{1}{2}\partial_{zz}\big[\sigma_z(z)^2 m_t\big], \tag{107}$$

with the same boundary conditions as in the stationary problem. As in standard solutions for discrete-time heterogeneous agent models, the distribution runs forward from an initial condition, while the HJB runs backward from a terminal condition. The initial distribution is not the 1984 distribution itself but a new one after the revaluation of wealth at the announcement, m_{0^+}, described below. Hand-to-mouth households at $a = 0$ form a mass point whose size now changes over time; on the grid it is part of the mass at the first wealth node. Since the income process does not change, the income marginal of m_t stays at its invariant distribution, and aggregate efficiency units of labor remain $\int\!\!\int z\, \mathrm{d}m_t = 1$.

Firms The static block remains unchanged except for the time-varying markup. With $L^P = 1 - \phi$,

$$Y_t = K_t^{\alpha}(L_t^P)^{1-\alpha}, \qquad w_t = \frac{1-\alpha}{\mu_t}\frac{Y_t}{L_t^P}, \qquad MRPK_t = \frac{\alpha}{\mu_t}\frac{Y_t}{K_t}. \tag{108}$$

The firm maximizes the present value of dividends, now discounted at the time-varying expected equity return,

$$\max \int_0^\infty \exp\Big[-\int_0^t (r_{R,s}-g)\,\mathrm{d}s\Big] DIV_t\,\mathrm{d}t, \qquad DIV_t = Y_t - w_t - I_t - (r_{S,t}-g)D_t + \dot{D}_t, \tag{109}$$

subject to the capital law of motion $\dot{K}_t = I_t - (\delta+g)K_t$ and the collateral constraint $D_t \le \lambda K_t$. The current-value Hamiltonian is the same as in equation (23) with time-varying prices. Solving the problem, we get familiar equations with time-varying prices:

$$MRPK_t = \lambda r_{S,t} + (1-\lambda) r_{R,t} + \delta. \tag{110}$$

The premium is positive in equilibrium, so the collateral constraint binds: $D_t = \lambda K_t$. Along the transition, capital is a state variable inherited from the past, and (110) determines the equity return given capital and the safe rate:

$$r_{R,t} = \frac{MRPK_t - \delta - \lambda r_{S,t}}{1-\lambda}. \tag{111}$$

Capital cannot jump at the announcement. So K_0 and D_0 are fixed. Investment, which enters the Hamiltonian linearly, leaves the firm indifferent once $q_t = 1$.

$$I_t = \dot{K}_t + (\delta+g)K_t, \tag{112}$$

Investment is determined in equilibrium by market clearing. We allow I_t to be of either sign and verify that it stays positive along the computed path, so that the no-divestment constraint never binds.

Equity value. The steady-state formula $V^E = DIV/(r_R - g)$ needs to be replaced by a standard asset-pricing equation

$$r_{R,t} = g + \frac{DIV_t}{V_t^E} + \frac{\dot{V}_t^E}{V_t^E}, \qquad \text{so} \qquad V_t^E = \int_t^\infty \exp\Big[-\int_t^s (r_{R,u}-g)\,\mathrm{d}u\Big] DIV_s\,\mathrm{d}s \tag{113}$$

under the no-bubble condition $\exp\big[-\int_t^s (r_{R,u}-g)\mathrm{d}u\big]\, V_s^E \to 0$. Along the transition part of the equity return is a capital gain, so the dividend yield and $r_{R,t}-g$ differ. Using $D_t = \lambda K_t$ and (110), the present value can be rewritten as book equity plus the present value of rents:

$$V_t^E = (1-\lambda)K_t + \int_t^\infty \exp\Big[-\int_t^s (r_{R,u}-g)\,\mathrm{d}u\Big]\Big[\Big(1-\frac{1}{\mu_s}\Big)Y_s - w_s\phi\Big]\mathrm{d}s. \tag{114}$$

Market equity therefore differs from book equity exactly by the capitalized markup rents net of overhead, both along the transition and in steady state.

Equity revaluation at announcement. Let 0^- denote the instant before the announcement, when the economy is in its 1984 steady state, and 0^+ the instant after. At $t = 0$ the announced markup path changes

the value of equity from $V_{0^-}^E$ to V_0^E, given by (113) on the new path. By (114), since K_0 is predetermined, this capital gain is exactly the revaluation of future markup rents. Households enter date 0 with their steady-state portfolios. Safe debt is instantaneous and priced at par, so it is not revalued. We assume all households get a capital gain proportional to the ex ante value of their equity $a_{R,0^-}(a,z)$. The revaluation happens with the share price $V_0^E/V_{0^-}^E$, so a household's equity holding $a_{R,0^-}(a,z)$ jumps by the same factor. Each household's wealth jumps to

$$a_{0^+} = a + \left(\frac{V_0^E}{V_{0^-}^E} - 1\right) a_{R,0^-}(a,z), \tag{115}$$

after which the households instantaneously re-optimize their portfolio holdings at given prices. The initial condition of the KFE is the 0^- distribution, with each household moved to its revalued wealth. Since equity is held mostly by wealthy households, the revaluation redistributes towards higher levels of wealth. This is necessary for equilibrium: without it, household wealth at 0^+ would still equal $D_{0^-} + V_{0^-}^E$, while the supply of assets would be $D_0 + V_0^E$, and the two asset markets could not both clear.

Market clearing and welfare. The safe-asset, equity, labor and goods markets clear at every date,

$$\iint (a - a_{R,t})\,\mathrm{d}m_t = D_t, \qquad \iint a_{R,t}\,\mathrm{d}m_t = V_t^E, \qquad \iint z\,\mathrm{d}m_t = 1, \qquad Y_t = C_t + I_t + \Psi_t, \tag{116}$$

where C_t and Ψ_t aggregate consumption and holding costs over m_t. Welfare is the consumption-equivalent variation of entering the transition at revalued wealth, relative to the initial steady state,

$$CEV^{tr} = \left(\frac{\iint V_0\big(a + (\kappa_0 - 1)\,a_{R,0^-}(a,z),\, z\big)\,\mathrm{d}m_{0^-}}{\iint V_{0^-}\,\mathrm{d}m_{0^-}}\right)^{\frac{1}{1-\gamma}} - 1 = \left(\frac{\iint V_0\,\mathrm{d}m_{0^+}}{\iint V_{0^-}\,\mathrm{d}m_{0^-}}\right)^{\frac{1}{1-\gamma}} - 1. \tag{117}$$

Along the path, CEV_t replaces the numerator with $\iint V_t\,\mathrm{d}m_t$.

Transition equilibrium. Given the announced markup path and K_0, a transition equilibrium is a path of prices $\{r_{S,t}, r_{R,t}, w_t\}$, firm choices $\{K_t, I_t, D_t, DIV_t\}$, equity values $\{V_t^E\}$, the revaluation $\frac{V_0^E}{V_{0^-}^E}$, and household value functions, policies and distributions $\{V_t, c_t, a_{R,t}, m_t\}$ such that:

- households optimize given prices;
- firms optimize and equity is priced by (113);
- the distribution evolves by the KFE from the revalued initial distribution;
- all markets clear;
- the economy converges to the terminal steady state.

Computation. We solve on $[0, T]$ on a time grid $0 = t_0 < \cdots < t_N = T$, $\Delta t_n = t_{n+1} - t_n$, with steps of at most one year while the markup moves and geometrically growing steps afterward. For the 1984–2020 path we use $T = 336$ years and $N = 120$. At T we impose the terminal steady state's value function, equity value and capital. The terminal distribution is not imposed: it is the endpoint of the KFE, and the remaining market gaps at T, below 10^{-3}, confirm that the horizon is long enough.

The unknowns are the safe rate at $t_0, \ldots, t_{N-1}$ and capital at $t_1, \ldots, t_{N-1}$. Given a guess, one evaluation of the equilibrium conditions proceeds as follows.

1. *Firms.* Compute Y_n, w_n and $MRPK_n$ from (108), $r_{R,n}$ from (111), $I_n = (K_{n+1} - K_n)/\Delta t_n + (\delta + g)K_n$, $D_n = \lambda K_n$, and DIV_n.

2. *Equity, backward.* From the terminal value, $V_n^E = \left(V_{n+1}^E + \Delta t_n \, DIV_n\right) / \left(1 + \Delta t_n (r_{R,n} - g)\right)$, the implicit discretization of (113).

3. *Revaluation.* $\frac{V_0^E}{V_{0^-}^E}$. Each node of the initial distribution is moved to its revalued wealth (115), and its mass is split between the two neighboring grid points so as to preserve mass and mean wealth. V_0 is evaluated at revalued wealth with the same weights to compute (117).

4. *HJB, backward.* From $V^N = V^\infty$, compute date-n policies from V^{n+1} by upwinding and solve the implicit step $\left[(1/\Delta t_n + \beta)\mathbf{I} - \mathbf{A}_n\right] V^n = u(c_n) + V^{n+1}/\Delta t_n$, where $\mathbf{A}_n$ is the generator matrix at date-n policies.

5. *KFE, forward.* Let m^n denote the vector of masses of m_{t_n} at the grid nodes, stacked in the same order as V^n. Starting from the revalued distribution $m^0 = m_{0+}$, $\left(\mathbf{I} - \Delta t_n \mathbf{A}_n^\top\right) m^{n+1} = m^n$. Because the KFE uses the transpose of the HJB generator, the two equations are discretized consistently, and since the rows of $\mathbf{A}_n$ sum to zero, total mass is conserved exactly.

6. *Residuals.* Safe-market excess demand at $t_0, \ldots, t_{N-1}$ and equity-market excess demand at $t_1, \ldots, t_{N-1}$, scaled by market size. The equity residual at t_0 is implied by the safe residual, as explained above; we report it as a check.

This gives a square system of $2N - 1$ equations in $2N - 1$ unknowns, $x = \left(\log(1 + r_{S,n})\right)_{n=0}^{N-1}$ and $\left(\log K_n\right)_{n=1}^{N-1}$. We solve it with a damped Newton method, $x^{(k+1)} = x^{(k)} + \lambda_k \, d^{(k)}$ with $J^{(k)} d^{(k)} = -H(x^{(k)})$, where H stacks the residuals and $J^{(k)}$ is the Jacobian of the full nonlinear system at the current iterate, computed by finite differences. When the Newton step is large, it is replaced by a Levenberg–Marquardt step of bounded size, and λ_k is reduced by backtracking until $\|H\|_2$ falls sufficiently. Every evaluation of H solves the full nonlinear HJB and KFE, so the solution is the nonlinear transition, not a linear approximation. We stop when the largest residual is below 10^{-7}, and reject any iterate at which the premium or investment is not positive.

Appendix B Model's Comparative Statics and Identification

Figure 14 reports comparative statics with respect to the six structural forces in the model. For each parameter, the left panel shows the equilibrium safe and risky returns, the middle panel shows the labor and profit shares, and the right panel reports welfare, measured as a consumption-equivalent welfare index normalized to 100 at the benchmark.

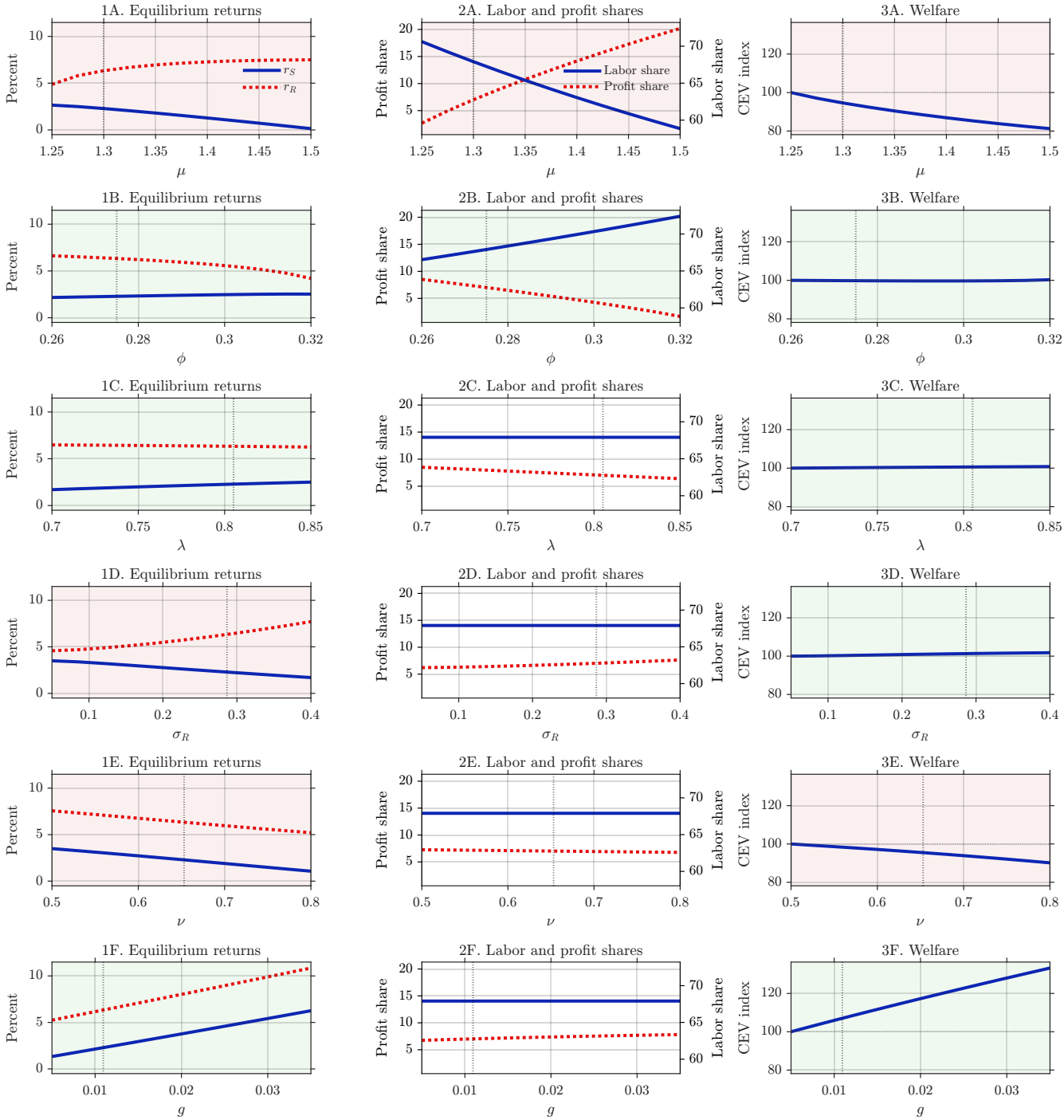


Figure 14: Comparative statics over six parameters of the model $\{\mu, \phi, \lambda, \sigma_R, \nu, g\}$.

Comparative statics on fixed costs. An increase in fixed costs ϕ, paid in labor units, raises the safe rate r_S and lowers the return on equity r_R, compressing the equity premium. At the same time, the labor share rises and the profit share declines sharply. Welfare is nearly flat, with at most a very small decline.

The reason is that higher fixed costs reduce the amount of profit income available to equity holders and redirect income toward labor. This lowers the importance of risky equity income in household portfolios and weakens precautionary demand for safe assets, thereby putting upward pressure on the safe rate. Since profits and dividends fall, the return required on equity also falls. In this sense, fixed costs work primarily through redistribution from profits to labor rather than through changes in markups or financial risk.

Comparative statics on the collateral constraint. A relaxation of the firms' collateral constraint, captured by a higher λ, raises the safe rate r_S and slightly lowers the return on equity r_R, thereby compressing the spread $r_R - r_S$. In contrast, the labor and profit shares remain essentially unchanged. Welfare increases modestly.

The main effect of a higher λ is to expand the supply of safe debt that firms can issue against collateral. As safe claims become more abundant, the equilibrium safe rate rises. At the same time, households require slightly less compensation for holding risky equity, so r_R declines somewhat. Since the collateral constraint mainly affects firms' financing choices rather than product-market power, its effect on factor shares is negligible. The increase in welfare reflects the reduction in financing distortions and the cheaper capital financing.

Comparative statics on equity-return volatility. Higher equity-return volatility σ_R lowers the safe rate r_S and raises the return on equity r_R, producing a sharp increase in the equity premium. The labor and profit shares are nearly unchanged. Welfare rises slightly in the figure.

The mechanism is straightforward. A higher σ_R increases the riskiness of equity holdings and therefore strengthens households' demand for safe assets as a hedge, pushing down r_S. At the same time, equity holders must be compensated for bearing more return risk, so r_R rises. Because this parameter affects financial risk rather than firms' pricing behavior, the labor and profit shares are largely unaffected. In our calibration, the increase in the expected return on equity slightly outweighs the direct adverse effect of higher volatility on average welfare, so welfare rises modestly, although the quantitative effect is small.

Comparative statics on labor-income risk. Higher labor-income risk, measured by the cross-sectional dispersion parameter ν, lowers both the safe rate r_S and the return on equity r_R, while leaving the equity premium roughly stable. The labor and profit shares remain essentially unchanged. Welfare declines substantially.

A rise in ν increases uninsurable background risk and strengthens precautionary saving motives. Households therefore demand more safe assets, which depresses the safe rate. Since higher precautionary saving also increases aggregate wealth accumulation and puts downward pressure on equilibrium

returns more broadly, the return on equity falls as well. Because ν affects the household income process rather than the distribution of production income between labor and profits, factor shares remain largely unchanged. The welfare decline is large, reflecting the direct cost of greater uninsurable labor-income risk.

Comparative statics on growth. An increase in productivity growth g raises both the safe rate r_S and the return on equity r_R, with a somewhat larger increase in the risky return. The labor and profit shares remain essentially unchanged, while welfare rises strongly.

This result follows directly from the equilibrium return decomposition. Higher growth raises the intertemporal benchmark for asset returns, shifting up both the safe and risky rates even in the absence of changes in markups or financial risk. Since growth does not directly alter the degree of market power or the division of income between labor and profits, the labor and profit shares remain approximately constant. The large rise in welfare reflects the fact that faster productivity growth improves consumption possibilities over time and raises the value of future income.

B.1 Identification

Table 4: Identification

Parameter	**Moments**					
	c.w. $\hat{\mu}$	Firm debt over assets	π^{acc}/Y	$\text{sd}(\log w^{disp})$	premium	g
$\nearrow$ μ	+	0	+	0	+	0
$\nearrow$ ϕ	0	0	−	0	−	0
$\nearrow$ λ	0	+	−	0	+/−	0
$\nearrow$ σ_R	0	0	0	0	+	0
$\nearrow$ ν	0	0	0/−	+	0	0
$\nearrow$ g	0	0	0	0	+/0	+
Data (1984–2020)	+	+	+	+	+	−

Table 4 summarizes the comparative statics underlying the six structural parameters. The identification is local around the mid-point of the minimum and the maximum of each parameter estimate in the data, and follows De Loecker et al. (2021). The table shows that changes in each parameter generate a distinct pattern of responses across the six targeted moments. The table also shows that, while some parameters uniquely identify a moment in the data, we also observe a rich set of interactions between the parameters through the general equilibrium forces. Consequently, the joint response of the targeted moments identifies the six structural parameters, since no two parameters generate the same comparative-static pattern across moments.

Appendix C Household heterogeneity: model vs data

Mian, Straub, and Sufi (2021) (MSS) emphasize how heterogeneity in saving rates can affect the natural rate: redistribution of income toward households with higher saving rates raises aggregate savings and puts downward pressure on their r^*. In Figure 15, we plot our model-implied saving rates over the income distribution for the initial and last periods of our sample and compare them with MSS statistics from the Survey of Consumer Finances. Our model matches the data qualitatively well, although it slightly undershoots saving rates at the top 10% of the income distribution. Standard heterogeneous-agent models struggle to even capture our qualitative pattern. The key is that, in our model, the equity cost function fades with wealth, so rich households obtain a higher net-of-cost return on wealth, as observed in Xavier (2021). An alternative to our cost function that would also yield such heterogeneous savings rates is wealth-in-utility, as in Straub (2019). However, it would be much harder to match the distribution of portfolio shares, especially for households near the borrowing constraint.

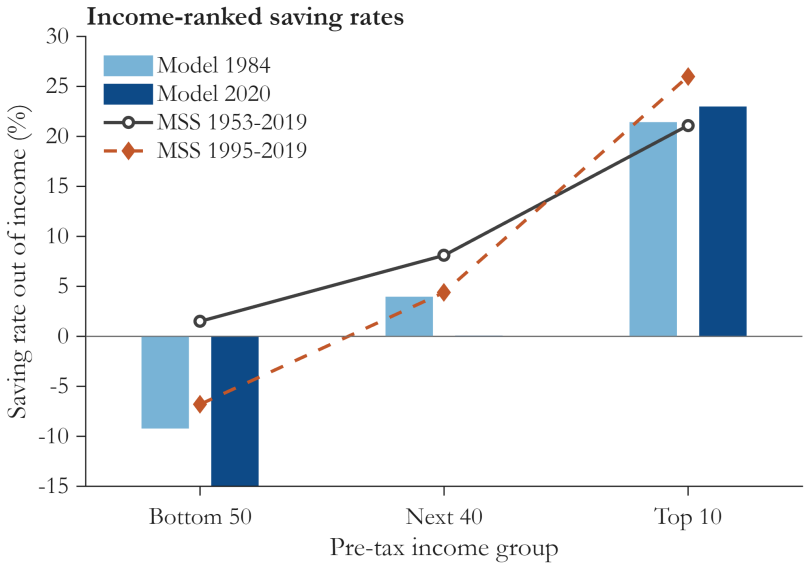


Figure 15: Saving rates along the income distribution.

Note: Model saving rates are computed from the stationary distributions of the estimated 1984 and 2020 economies. Pre-tax cash income includes labor income, safe interest income, and equity payouts, but excludes equity valuation gains. Saving is cash income minus consumption and equity-holding costs. Households are ranked by cash income and divided into groups containing exactly 50, 40, and 10 percent of the population. Each reported saving rate is aggregate group saving divided by aggregate group income. MSS statistics are from Mian et al. (2021) and rank households by income within birth cohorts. The MSS series are averages over the indicated sample periods and therefore do not correspond directly to the model years.

Another important aspect of our model is how portfolio shares are distributed across wealth levels and how they evolve over time, since richer households tend to benefit more from increased market power by holding higher equity shares. In Figure 16, we plot the portfolio shares of risky equity for the 25th, 50th, 90th, and 99th percentiles of the wealth distribution in our model and compare them with those of the Survey of Consumer Finances (SCF). Even though none of these moments are targeted year by year, we match well the evolution of most of the portfolio shares, except for those in the 90th percentile, which we overshoot. Figure 17 explains the intuition for why this happens: we plot the

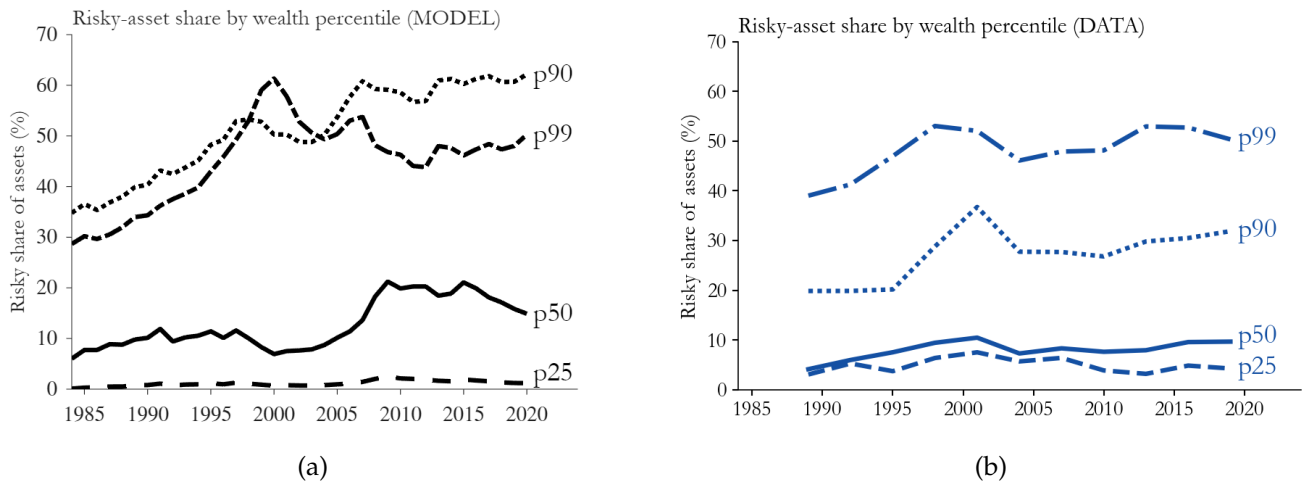


Figure 16: Portfolio shares of risky assets by selected percentiles. Model fit vs Data. 1984-2020.

average risky portfolio share across the wealth distribution for our model calibration that matches 1984 data. In the absence of our equity-holding cost, the average risky share would monotonically decline towards the Merton share as wealth increases. However, our cost makes it more difficult for poorer households to hold equity, reducing their equity participation to realistic levels. As wealth increases, the equity cost per additional unit invested declines, and households hold higher shares of risky assets until around the 90th percentile, where the first effect reemerges, and the share begins to decline again towards the Merton share. Fixing this feature would require modeling the top 1% and the top 0.1% of the wealth distribution separately, for example, by adding an entrepreneurial state with a contractual constraint that prevents them from diversifying their portfolios, as in Chari et al. (2025). By doing so, we could still match the top 1% share, which matters for safe-asset demand, without overshooting the 90th percentile. However, we do not expect the quantitative results to change much.

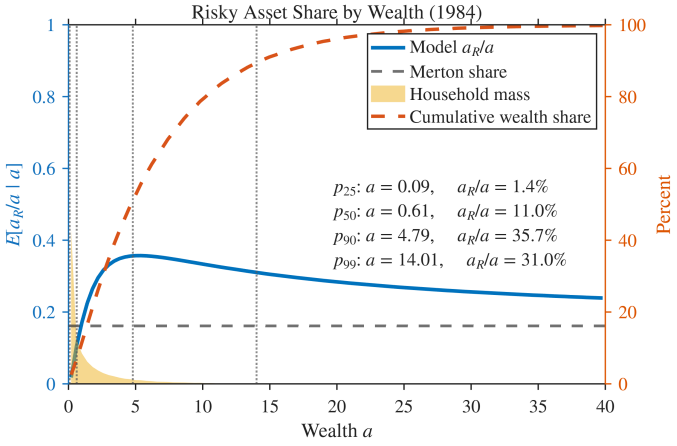
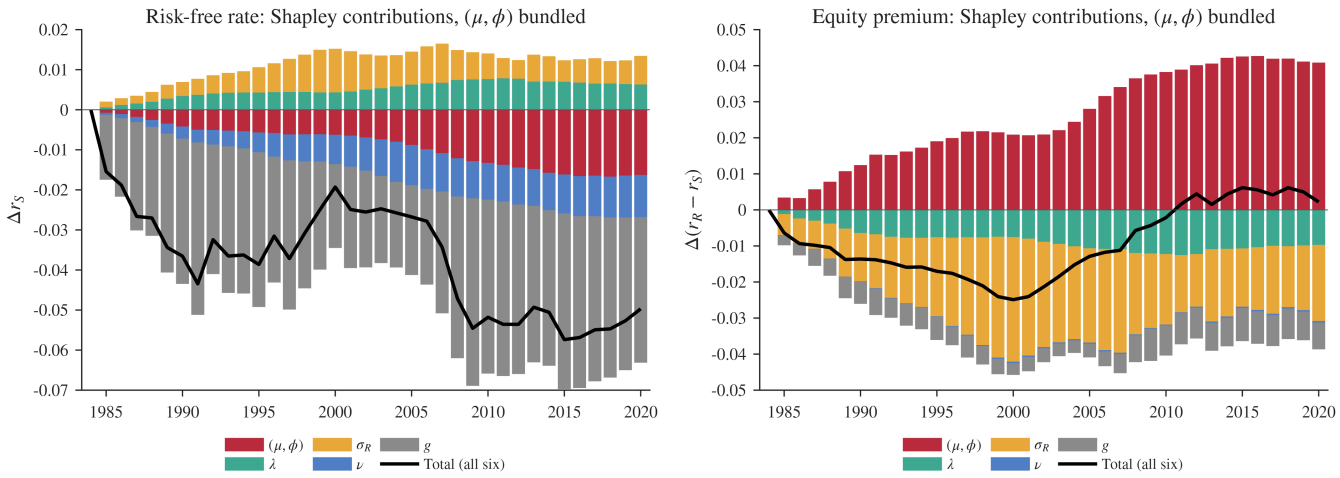


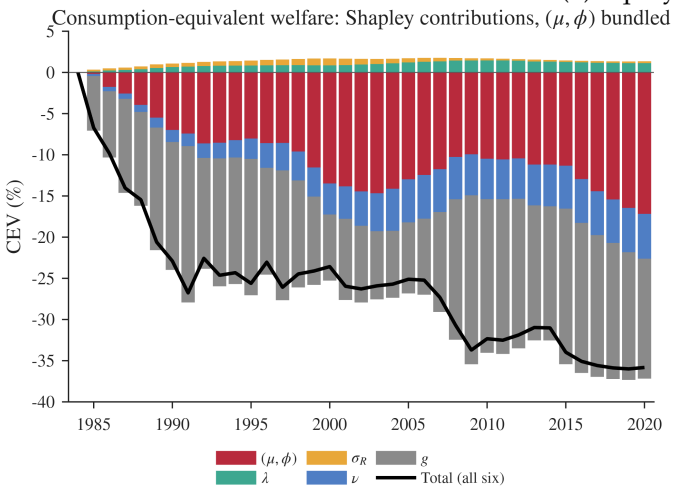
Figure 17: Average risky asset share over the wealth distribution.

Appendix D Extra Figures



(a) Risk-free rate

(b) Equity premium



(c) Welfare (CEV)

Figure 18: Contribution of different mechanisms to the counterfactual changes in the risk-free rate, equity premium, and welfare.

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